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Stability analysis of residual-affected HCCI using convex optimization

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ABSTRACT

Residual-affected homogeneous charge compression ignition (HCCI) is a promising methodology for simultaneously reducing emissions and fuel consumption. However, the process relies on cycle-to-cycle coupling between subsequent engine cycles through the exhaust gas temperature, resulting in sections of the state space which are unstable. This paper exploits a previously validated control model of HCCI to analytically determine the area of the state space which is stable to perturbation of either combustion timing or in-cylinder pressure. As efforts to control and expand the operating range of HCCI continue, analytical stability tools like that developed here will likely play an increasingly important role.

1. Introduction

Homogeneous charge compression ignition (HCCI) is an approach for increasing efficiency and reducing NO_x emissions in internal combustion engines. Improvements in efficiency of 15–20% compared to a conventional spark ignited (SI) engine are possible (Zhao et al., 2003), making HCCI efficiencies comparable to diesel engines. Unlike diesel combustion, however, the lack of fuel rich regions in HCCI results in little or no particulate emissions, a common issue with diesel strategies. Furthermore, the combustion of a homogeneous reactant mixture during HCCI leads to a reduction in the peak combustion temperature, lowering NO_x levels compared to conventional SI and diesel strategies. One effective strategy for achieving HCCI in engines with moderate compression ratios is through the reinduction (Caton, Simon, Gerdes, & Edwards, 2003; Law, Kemp, Allen, Kirkpatrick, & Copland, 2001) or trapping (Law et al., 2001) of residual exhaust gas via variable valve actuation (VVA). This methodology of using residual gas is called residual-affected HCCI.

As shown in Fig. 1, residual-affected HCCI via exhaust reinduction is achieved by using a flexible valve system to hold the intake and exhaust valves open during a portion of the intake stroke. This leads to the induction of both reactant (fuel and air) and residual (previously exhausted combustion products) gases from the intake and exhaust manifolds, respectively. Residual-affected HCCI can also be achieved by retaining some exhaust gas in the cylinder by closing the exhaust valve early during the exhaust stroke. The specific amounts of reactant and residual are varied through the modulation of the intake and exhaust valves. Following the induction process, the compression of the reactant/residual mixture results in the increase of both the in-cylinder mixture concentrations and temperature. If the reactant concentration and temperature reach sufficient levels, a uniform autoignition process occurs, resulting in a combustion-induced elevation of in-cylinder pressure and temperature. During the expansion stroke this elevated pressure is used to effectively push the piston, resulting in the extraction of useful work. The exhaust stroke then expels the hot combustion products into the exhaust manifold. Unlike conventional strategies, a portion of the exhausted gas is then reinducted or trapped for use during the subsequent engine cycle. This reinduction/trapping process couples engine cycles through the exhaust gas temperature.

An interesting characteristic of residual-affected HCCI is the self-stabilizing behavior of the process at some operating conditions. In particular the combustion timing, work output, and in-cylinder peak pressure often converge to a stable equilibrium point following modest perturbations in combustion timing ($\pm 1^\circ$) or in-cylinder pressure evolution (for instance, ± 5 atm on peak pressure). This behavior is due to the competing residual-induced heating and dilution of the reactant gas. For example, an increase in the amount of hot residual gas will elevate the temperature of the pre-compression reactant/residual mixture during the corresponding engine cycle. This leads to an acceleration of the chemical kinetics and causes the combustion event to occur earlier. The increased residual gas also dilutes the reactant gases, decreasing the amount of fuel and oxygen available for combustion. This leads to a decrease in the post-combustion temperature resulting in an accompanying decrease in the temperature of the residual available for the next engine cycle. Following several engine cycles these two effects (reactant heating and dilution) can balance, resulting in a stable steady state operating condition. However, large perturbations in

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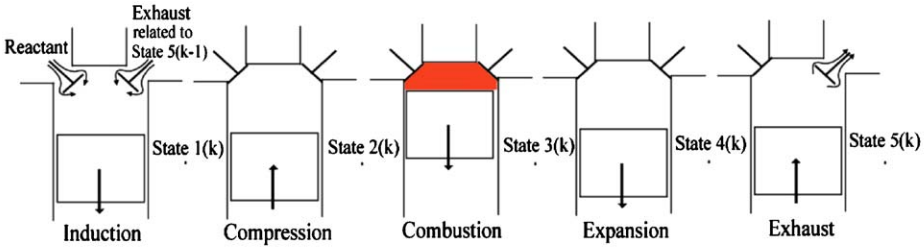

Fig. 1. General view of partitioned HCCI cycle with exhaust reinduction.

inducted gas composition, combustion timing or exhaust gas temperature (which may occur during large step changes in HCCI operating condition or mode transition from SI to HCCI) often lead to de-stabilization of residual-affected HCCI. Understanding the stability limits of residual-affected HCCI, both with and without active feedback control, is a fundamental problem in making HCCI a practical combustion strategy.

In earlier work (Shaver, Roelle, Gerdes, Caton, & Edwards, 2005) a 10-state simulation model (developed to build intuition, as well as provide an accurate virtual testbed) was shown to exhibit the behavior seen during changes in inducted gas composition in an experimental testbed. A simplified 2-state control model (constructed as a means to directly synthesize control strategies (Shaver, Roelle, & Gerdes, 2008) also shows this self-stabilizing behavior. Utilizing a physics-based approach, the control model, like the 10-state model, is capable of capturing the self-stabilizing behavior because it explicitly includes the effect that residual modulation has on the pre-compression and post-combustion temperatures, as well as their role in determining the timing of the combustion event via the chemical kinetics and cycle-to-cycle coupling through the exhaust gas temperature.

An additional benefit of the control model, relative to the more complex simulation model, is its amenability to stability analysis via some recently developed tools, in particular the application of sum of squares (SOS) programming to Lyapunov stability approaches. This paper outlines an approach for showing Lyapunov stability about an operating point (defined by perturbations in combustion timing and peak in-cylinder pressure) by exploiting the structure of the control model dynamics (rational vector fields) to convert the stability problem into a tractable convex optimization program. This approach is implemented to prove that the dynamics described by the 2-state control model are stable to modest perturbations of either of the two model outputs, the combustion timing or peak in-cylinder pressure. The resulting region of attraction proves stability of the system over the region of interest, whereas an analysis depending on simulation results alone may miss complex dynamics associated with certain portions of the state space.

These results provide an analytical approach for studying the self-stabilizing behavior of residual-affected HCCI often witnessed in experiment. This framework also provides a means to assess the stability of the non-linear HCCI dynamics in closed-loop with candidate control strategies, subject matter for future work.

2. Modeling approach

The dynamic coupling between VVA-controllable inputs (inducted gas composition and effective compression ratio) and measurable outputs of the residual-affected HCCI process (combustion timing and peak pressure) is captured with a 2-input 2-output control model (Shaver et al., 2008). The framework for developing the control model is to partition the engine cycle into five stages, as shown in Fig. 2:

1. mixing of reactant and re-inducted product gases during a constant pressure, adiabatic induction process;
2. isentropic compression to the point where combustion initiates;
3. constant volume combustion to major products with heat transfer;
4. isentropic expansion to the point where the exhaust valve opens; and
5. isentropic expansion through the exhaust valve.

The states shown in Fig. 2 represent the temperature and pressure of the in-cylinder gas. For instance, state 2 represents the temperature and pressure of the reactant gas post-compression, just before the constant-volume combustion event. State 3 represents the temperature and pressure of the combustion product gas immediately after combustion.

In the formulation the constant-volume combustion event is assumed to occur at or after the top dead center (TDC) position. This is a valid assumption for typical steady state HCCI operating conditions and modest perturbations around steady state—the conditions considered in this paper.

The re-inducted product temperature is directly related to the exhaust temperature from the previous cycle, and is modeled as such. The first model input is the inducted gas composition, formulated as the ratio of the moles of re-inducted product N_p to the moles of inducted reactant charge N_r, $\alpha \equiv N_p/N_r$. The second model input is the IVC valve timing, which dictates the volume, $V_1 = V(\theta_1)$, between the induction and the compression stages, determining start of compression and therefore the effective compression ratio. Model outputs are the peak pressure, P, and the volume at the constant volume combustion event, $V_{23} = V(\theta_{23})$, which acts as a proxy for combustion timing. By linking the thermodynamic states of the system together, a dynamic model of peak pressure, P, and combustion timing, θ_{23}, for residual-affected HCCI is formulated. Note that at points between stages, the cylinder volume (see Fig. 2) is either known or is a model output (as is the case for $V_{23} = V(\theta_{23})$).

In the model formulation the in-cylinder gas specific heat and specific heat ratio, γ, are assumed constant. In reality, both will have small variations as a function of gas composition and temperature. Applying the assumptions outlined above, as well as coupling one engine cycle to the subsequent one through the temperature of the reinducted exhaust gas allows the formulation of a 2-input 2-output control for residual-affected HCCI which has been validated during steady-state and transient operation (see Shaver et al., 2008 for more details), where the inputs are inducted gas composition and effective compression ratio, and the outputs are combustion timing and peak pressure; a key characteristic of the model in the coupling between engine cycles