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Solution of temperature distribution under frictional heating with consideration of material inhomogeneity

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ABSTRACT

Nonmetallic inclusions and defects can be commonly found in the initial metallurgical process of metal. This paper studies the influence of the size, position and interval of inhomogeneities on temperature field of inhomogeneous materials under frictional heating. A thermo-mechanical model of point contact is established based on the semi-analytical method, the equivalent inclusion method and three-dimensional high-order theory. The developed algorithm has good stability and convergence. The results show that the large inclusion size near the contact surface has significant effect on the temperature field. When the thermal conductivity of inhomogeneity is higher than that of the substrate, the temperature rise in the contact zone is lower; otherwise, opposite trend is observed.

1. Introduction

Nonmetallic inclusions often emerge in the smelting process of steel. The elastic modulus of the inhomogeneity and matrix may be several times different, which may affect contact pressure and stress distribution, and consequently affect the performance of mechanical elements on the vibration, fatigue life and so on. A series of work has been devoted to the calculation of inclusion problem; however, it is difficult to find analytical solution due to the discrepancy in material properties and complex shaped inclusions. The problem with material dissimilarity between matrix and inhomogeneities can be solved by the Equivalent Inclusion Method (EIM) proposed by Eshelby [1], which treats inhomogeneous inclusions as equivalent inclusions with additional equivalent eigenstrains; then the inclusion problem is converted into the solution of elastic field caused by eigenstrains. The problem with complex shaped inhomogeneities has to be solved by means of numerical methods. The numerical solutions are early based on the solutions given by Chiu [2] for cuboidal inhomogeneity. Zhou et al. [3,4] obtained the elastic field of infinite space and half space containing multiple inhomogeneities with arbitrary shape, and later followed by the work by Liu et al. [5] and Wang et al. [6]. Liu et al. derived the general influence coefficients that relate the eigenstrain to displacement and stress in a half space based on Galerkin vectors, and speeded up the solution by the Fourier transform-based algorithm; while Wang et al. proposed a simple but efficient method to solve the stresses caused by inhomogeneities in a half space based on full space solutions. Based on the above-mentioned methods and solutions, Zhang et al. [7] studied the effect of the inhomogenieties on elastohydrodynamic lubrication performance. Wang et al. [8] showed that the inhomogeneity in bearing races can induce large surface distributed displacements, which has great effect on dynamic behavior of rolling bearing.

Usually, the inhomogenieties and matrix have much different thermal conductivity, which would greatly change the thermal conductivity properties of composite materials; consequently, the temperature field of inhomogeneous materials may be greatly different from the homogeneous materials. Many studies have investigated the effects of inhomogeneity on the thermal properties of composites materials. On the aspect of experiments, Zhou et al. [9] used different amount, particle size and surface treatment of Al_2O_3 fillers to fill silicone rubber, and studied the effect of these variables on the thermal conductivity and mechanical properties. Bartczak et al. [10] revealed that the thermal conductivity of composites was determined by the average inter-inclusion distance. When the distance between inhomogeneities become smaller, there will be more chances to form thermal conductive 'pathway'. Wei et al. [11] further discussed the effect of hybridized inclusions on thermal conductivity of polymer composites. On the theoretical side, much work has been devoted to the calculation of the heat transfer and temperature field in composites materials, among which the higher-order theory is an accurate method for analysis of materials in the micro-scale [12]. However, when huge amounts of inclusions or large thermal gradients around the inclusion/matrix interface is involved, fine mesh is needed and this method becomes time consuming. To solve this problem, Bansal et al. [13]

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Nomenclature

a	Hertz contact radius
E	Young's modulus
ν	Poisson's ratio
$\boldsymbol{\sigma}^{e}$	Stress caused by contact pressure
$\boldsymbol{\sigma}^{P}$	eigenstress caused by initial eigenstrain
$\boldsymbol{\sigma}^{*}$	eigenstress caused by equivalent eigenstrain
$\boldsymbol{\varepsilon}^{P}$	equivalent eigenstrain
$\boldsymbol{\varepsilon}^{*}$	initial eigenstrain
c_x, c_y, c_z, c	lengths of a cubic inhomogeneity
$\boldsymbol{C}$	elastic moduli of the matrix material
$\boldsymbol{C}^{I}$	elastic moduli of the inhomogeneity
$\boldsymbol{P}$	contact pressure
$\boldsymbol{Q}$	Heat flux vector
$T^{(\alpha,\beta,\gamma)}$	temperature field in the subcell (α,β,γ)
$T_{(mnl)}^{(\alpha,\beta,\gamma)}$	coefficients associated with high-order terms in the temperature field expansion with in the subcell (α,β,γ)
$d_{\alpha}^{(\alpha,\beta,\gamma)}$, $h_{\beta}^{(\alpha,\beta,\gamma)}$, $l_{\gamma}^{(\alpha,\beta,\gamma)}$	length of the subcell (α,β,γ)
$k_i^{(\alpha,\beta,\gamma)}$	coefficients of thermal conductivity in the subcell (α,β,γ)

employed the surface-averaged temperatures to replace the temperature expressed by the second-order polynomial expansion in original high order theory.

For homogeneous materials or function graded materials (FGMs), the thermoelastic contact problems have been extensively investigated. Choi and Paulino [14] investigated the steady-state thermoelastic contact with the assumption that the effect of convection could be ignored compared with the conduction at the slow sliding speed. Barik et al. [15] investigated the effect of the graded parameters on surface temperature and contact pressure. Liu et al. [16] studied the effects of different gradient index, Peclet number and friction coefficient on the contact pressure and temperature distribution of surfaces. However, there lack the researches about the thermoelastic contact problem of composites materials at micro-level. As most elements operate in mixed or boundary lubrication, where one of the important issues is the asperity contact subjected to the heat transfer across the contact interfaces. It can be expected that the variation of the temperature in the contacting bodies will cause the change of contact conditions, and therefore, the contact pressure will vary with the frictional heating due to thermal deformation and asperity distortion. At the same time, the variation of temperature rise will cause the thermal stress, which may further intensify the stress concentration induced by the inhomogeneities, especially at the interfaces between inhomogeneity and matrix. Consequently, it will significantly affect the fatigue life of the components. Therefore, the solution of temperature rise in the contact region subjected to frictional heating is fundamental and essential for the realistic modeling of the performance of mechanical components. In this paper, the three-dimensional solution of temperature field of inhomogeneous material under sliding friction conditions is investigated. Firstly, the contact pressure is solved using the semi-analytical method and the EIM theory, and the frictional heating is obtained based on the contact pressure distribution; then the temperature field is obtained by solving the steady-state heat transfer equations based on the improved high-order theory. By changing the parameters such as the inclusion size, depth and material properties, their effects on the temperature field of composites materials are investigated.

2. Theoretical models and approaches

2.1. Contact model with considering inhomogeneity

The contact between rigid ball and half space containing inhomogeneities is modeled and solved based on minimizing the total complementary potential energy to get the contact pressure.

The gap between two contact surfaces can be expressed as follows:

$$h(x_1, x_2) = h_i(x_1, x_2) + \delta + u_e(x_1, x_2) + u^*(x_1, x_2) \tag{1}$$

where h_i is the initial geometric gap between two contact surfaces; δ is rigid body approach; u_e is the surface normal elastic displacement induced by contact pressure, which can be solved based on the well-known Boussinesq integral [17]; u^* is the eigendisplacement induced by the equivalent eigenstrain and initial eigenstrain, which can be calculated as follows:

$$\boldsymbol{u}^*(x_1, x_2) = \sum_{\varphi=1}^{N_3}\sum_{\zeta=1}^{N_2}\sum_{\xi=1}^{N_1} \mathbf{S}_{\xi-\alpha,\zeta-\beta,\varphi}(\varepsilon^*_{\xi,\zeta,\varphi} + \varepsilon^p_{\xi,\zeta,\varphi}) \tag{2}$$

where $\mathbf{S}$ is the influence coefficients that relate the eigenstrain to surface normal displacement, its explicit expression can be found in Ref. [5], while the equivalent eigenstrain is related to the inhomogeneity contained in half space and can be obtained as shown in the next section.

The gap h (x_1,x_2) and contact pressure P (x_1,x_2) satisfy the Kuhn-Tucker complementary conditions, namely,

$$\mathrm{h(x1,x2)*P(x1,x2)} = 0 \tag{3}$$

with the constraint conditions of h $(x_1,x_2)\geq 0$ and P $(x_1,x_2)\geq 0$. In contact region, $P > 0$, $h = 0$; out of contact region, $P = 0$, $h > 0$.

Further, the integration of contact pressure should balance with the applied load,

$$\iint_{\Omega} PdXdY=W \tag{4}$$

Eqs. (1)–(4) formulate the contact problem between rigid body and half space containing inhomogeneity. The contact pressure, contact area and eigenstrains, etc. can be obtained by solving this set of equations. The solution method is based on minimizing the total complementary potential energy, which can be transferred into an equivalent problem that obeys the Kuhn-Tucker complementary conditions. Then the conjugate gradient method (CGM) is used to efficiently search the contact area and pressure distribution P (x_1,x_2), where the calculations related to the deformations u_e and u^* are speeded up by 2D FFT algorithm. For more details about the algorithms, the readers may refer to Refs. [18,19] for 2D FFT algorithm and Ref. [20] for the iteration scheme of CGM. If the relative error $|P\text{-}P_{\text{old}}|/P$ is smaller than 10^{-5}, the iteration process is stopped.

2.2. The solution of equivalent eigenstrain based on equivalent inclusion method

In Fig. 1, assuming that the matrix of half space has the elastic tensor of C_{ijkl} and the inhomogeneities have the elastic tensor of $\boldsymbol{C}^{I}_{ijkl}$. According to EIM method [1], the inhomogeneity can be converted into the inclusion with the same elastic tensor as matrix but being subjected to an extra equivalent eigenstrain $\boldsymbol{\varepsilon}^*_{ij}$. In addition, initial eigenstrain $\boldsymbol{\varepsilon}^p_{ij}$ generally exists in inhomogeneities. Based on EIM and Hooke's law, controlling equation as Eq. (5) can be derived to solve equivalent eigenstrain $\boldsymbol{\varepsilon}^*_{ij}$.

$$\boldsymbol{C}^{I}_{ijkl}\boldsymbol{C}^{-1}_{klmq}\boldsymbol{\sigma}^*_{mq} - \boldsymbol{\sigma}^*_{ij} + \boldsymbol{C}^{I}_{ijkl}\boldsymbol{\varepsilon}^*_{kl} = \boldsymbol{\sigma}^p_{ij} + \boldsymbol{\sigma}^e_{ij} - \boldsymbol{C}^{I}_{ijkl}\boldsymbol{C}^{-1}_{klmq}(\boldsymbol{\sigma}^p_{ij} + \boldsymbol{\sigma}^e_{ij}) \tag{5}$$

Where $\boldsymbol{\sigma}^e_{ij}$ is the stress caused by the contact pressure directly, its solution can be obtained based on the influence coefficients such as given in Refs. [6,21]. $\boldsymbol{\sigma}^*_{ij}$ and $\boldsymbol{\sigma}^p_{ij}$ are eigenstresses that are induced by equivalent eigenstrains and initial eigenstrains, respectively. $\boldsymbol{\sigma}^*_{ij}$ can be calculated based on the following formula, and $\boldsymbol{\sigma}^p_{ij}$ by the same method