Tribology International 119 (2018) 531–538

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

# Tangential contact analysis of spherical pump based on fractal theory

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A R T I C L E I N F O

*Keywords:*
Tangential contact stiffness (TCS)
Analytical method
Fractal theory
Spherical pump

A B S T R A C T

The tangential contact between piston and cylinder in spherical pump is investigated using both analytical method and fractal theory. The analytical model is established firstly, friction coefficient between piston and cylinder is considered. Effects of operating conditions and structural parameters are analyzed. Besides, a fractal contact model for spherical pump is proposed, and the model is validated by Jiang's general contact model and experimental results. Variation of the tangential contact stiffness with factors such as fractal dimension, characteristics length, friction coefficient, material hardness, yield stress, elastic modulus and loaded force are studied comprehensively. Results indicate these parameters have complex effects on tangential contact stiffness under different operating conditions, the related conclusions can also be utilized to analyze the tangential contact characteristics of spherical surfaces.

## 1. Introduction

Spherical pump [1] has the advantages of compact, low vibration and noise emission. As illustrated in Fig. 1, it has a spherical piston and a related cylinder, working capacity and pressure can be improved due to the specific ball-in-socket structure. However, conforming contact between piston and cylinder enlarges contact area [2], which affects tribological behavior between piston and cylinder, such as scuffing and seizure. Therefore it is essential to investigate the tangential contact properties between piston and cylinder.

1-piston; 2-cylinder cover; 3-passage; 4-working chamber V1; 5-connection screw; 6-shaft; 7-shaft bracket; 8-rotary disc; 9-cylinder body; 10-central pin; 11-working chamber V2; 12/13-inlet/exhaust passage; Contact stiffness studies have been conducted many years based on different theories, such as the analytical method [2–5], fractal contact theory [6–14], statistic method [15–17], experimental approach [18–21], multi-scale method [22–24] and FEM [25], etc. Contact characterization has a significant effect on the physical phenomena such as friction, wear, adhesion, lubrication, damping, etc. Johnson gives outline of the traditional analytical method for tangential contact stiffness (TCS) in his book [3]. Many further studies are conducted, Nagatani [2] proposed two practical semi-analytical methods to analyze the contact pressure and shear stress on conforming contact problem. The proposed methods are validated by finite element simulation. Kogut [4] presented a static friction model for elastic-plastic contact rough surfaces, effects of plasticity index, adhesion coefficient, external force, nominal contact area on the static coefficient are investigated. Liu [5] developed a thermomechanical contact model to investigate the frictional heating.

Fractal theory is widely used in asperity contact analysis for decades. The first fractal contact model for real rough surfaces are proposed by Majumdar and Bushan [11,26] (MB model). Buzio [6] studied the mechanical response of self-affine thin films using a micrometric flat probe. They conclude that both reducing fractal dimension and surface roughness can strengthen the solid interface. Shi [7] investigated the TCS of rough cylindrical faying surfaces utilizing fractal theory, they defined a shape influence coefficient for the cylindrical contact. Results indicate the applied force, material properties, fractal dimension and surface shape have different effects on the TCS of faying surfaces. Zhang and Wang [8] established a tangential damping model to investigate the energy dissipation mechanism in contact surfaces using a modified MB model. Their work can be used to predict the damping effect of multi-layer structures in engineering field. Jiang [9] proposed a general contact stiffness model to investigate the contact between machined planes, which include milling, grinding and scraping processes. They also validated the model by experiments. Wang [10] deduced the equivalent normal stiffness and damping model for sliding joint by fractal theory combing with lubrication analysis, they found the ideal contact state by simulation and experiment. Liu [12] and Chen [14] considered the friction coefficient in normal contact analysis using fractal

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https://doi.org/10.1016/j.triboint.2017.11.034
Received 27 September 2017; Received in revised form 9 November 2017; Accepted 20 November 2017
Available online 22 November 2017

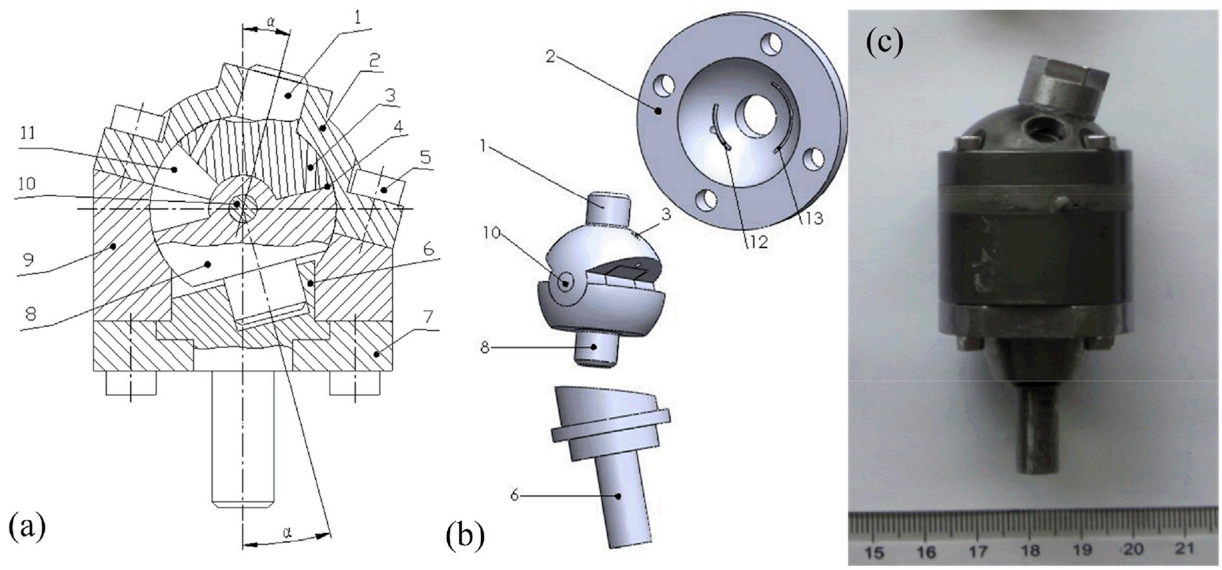


**Fig. 1.** Spherical pump (a) sectional schematic (b) 3D view (c) prototype [1].

theory. In fact, the widely utilized GW model [15] is based on the statistical theory. Hemispherical asperities distributed on the rough surface and represented by the root-mean-square curvature of the surface. Limitation of GW model is the curvature of hemispherical asperities is constant, which is over idealized and unrealistic. Wu [17] improved GW model and proved that asperity curvature can be estimated by root mean square curvature of profile. Kogut [16] made a contact modeling comparison between the statistical and fractal approaches. Besides the methods mentioned above, Tong utilized the molecular dynamics simulation to investigate the 2-D nanoscale sliding contacts of a textured surface at multiscale level. Jackson [22,23] also conducted the related multiscale contact model to predict the real contact area. Experimental manners are widely used and improved timely to verify the reasonability of each related methods.

***The main purpose of the present study*** is to investigate the TCS between piston and cylinder in spherical pump. Contributions of this paper are as follows:

- The analytical method is employed to investigate the TCS of spherical pump.
- Effects of operating conditions and structural parameters, such as normal (working pressure) and tangential force, piston radius and radial clearance, on TCS are investigated and discussed.
- Fractal tangential contact model of piston and cylinder are established, the model is validated by Jiang's general contact stiffness model and experiments [9].
- Effects of the influential parameters on TCS are studied and analyzed.

By utilizing the microscopic fractal theory and macroscopic analytical approach, we can establish the tangential contact model at different scales, to comprehensively study the deformation, friction and wear mechanism from different perspectives. Besides, effects of the pump's specific structural properties and operational conditions on tangential contact are also considered. The ultimate purpose of the present investigation is attempt to find the optimal design schemes for spherical pump, including the material and structural parameters selection, surface treatment approach, etc.

## 2. Tangential contact analysis

### 2.1. Equivalent ball-on-plane model

Schematic diagrams of the spherical pump are illustrated in Fig. 2(a) and (b). The spherical piston rotates around axis $z'$ and it is wrapped by a ball-socket cylinder cover. Radii of piston and cylinder cover are $R_p$ and $R_c$ respectively. An equivalent ball-on-plane model is utilized to simplify the conforming contact between the piston and cylinder as demonstrated in Fig. 2(b). The curvature and elastic modulus of the effective contact model are defined as

$$\frac{1}{R_{eff}} = \frac{1}{R_p} - \frac{1}{R_c} \tag{1}$$

$$\frac{1}{E_{eff}} = \frac{1-\nu_p^2}{E_p} + \frac{1-\nu_c^2}{E_c} \tag{2}$$

where $E_p$, $\nu_p$ and $E_c$, $\nu_c$ represent the moduli of elasticity and Poisson's ratios of the piston and cylinder.

From Fig. 2(a) we can observe that the radial clearance between piston and cylinder cover are denoted by$c_r$, which can be expressed as

$$c_r = R_c - R_p \tag{3}$$

Substituting Eq. (3) into Eq. (1), then gives the effective radius as

$$R_{eff} = R_p\left(1 + \frac{R_p}{c_r}\right) \tag{4}$$

When pump works, piston and cylinder cover surfaces are subjected to rather high pressure, which produces local deformations that cannot be neglected. For the equivalent ball-on-plane contact model, Hertzian theory for a static dry contact can be utilized to approximate the local deformation. The radius of the contact area is given by

$$a = \left(\frac{3FR_{eff}}{4E_{eff}}\right)^{\frac{1}{3}} \tag{5}$$

where $F$ denotes the normal force illustrated in Fig. 2(b).

### 2.2. Tangential contact analysis of the single model

The equivalent contact model between piston and cylinder without load is demonstrated in Fig. 3(a). In Fig. 3(b), a normal force $F$ is applied leading to the normal displacement $\delta$, radius of the contact region is denoted by $a$. Besides the normal force $F$, a tangential force $T$ which is parallel to the $x$ direction is loaded. The tangential force $T$ leads to the