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Adhesion of rigid rough contacts with bounded distribution of heights

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ABSTRACT

We develop a "Bradley" (rigid) model for a rough surface with bounded or non-bounded distribution of heights. We observe a large effect of the distribution of heights: for example, for Weibull distributions, the decay from the theoretical strength becomes an inverse power law with the roughness amplitude normalized by the adhesion interaction distance. For Gaussian surfaces which are in principle unbounded distributions, only weak dependence is found on the details of the roughness spectrum. If the truncation comes from a natural process like wear where the height distribution is squashed at a certain truncation level, the latter factor dominates.

1. Introduction

Bradley [5] gave the first solution for the adhesive force between rigid bodies (spheres), and computed it to be $2\pi R\Delta\gamma$, where $\Delta\gamma$ is the surface energy of the sphere, and R is radius: Johnson et al. [12] later included the effect of elasticity, while concentrating the adhesion forces to within the contact area, and surprisingly found just a minor difference with respect to the Bradley equation, with a prefactor 1.5 instead of 2, suggesting an independence on elastic modulus. Some debate emerged with another theory (DMT, [8]), which included elastic deformations but only corresponding to the compressive parts of the load (assuming the contact area was given by Hertzian theory). Interestingly, DMT obtained Bradley's result *exactly*—because at pull-off, the DMT theory suggests the contact area is zero, there is no compressive load transmitted. Finally, Tabor [21] made clear that the range of prefactors depend on a parameter which permits to see how the full solution including the Lennard–Jones 6–12 potential law moves from the JKR regime for high Tabor parameters, down to the DMT-Bradley at low values (one alternative solution to cover the transition being the Maugis–Dugdale one).

In the general case, however, there is no reason to expect the Bradley limit will correspond even remotely to the other possible theories (equivalent of DMT, JKR) since for the latter theories we expect a dependence on the elastic modulus which in turn will introduce dependence on many other factors. Indeed, both DMT and JKR depend (differently) on the elastic modulus for contact of cylinders for example, as shown by Baney and Hui [2].

For rough surfaces, Fuller and Tabor [10] proved within this approximation that adhesion is destroyed by a small amount of roughness, because of the competition between compression exchanged by the highest asperities, and the pulling forces exerted by the lowest ones. Other theories like Persson and Tosatti [16] or Persson [17], or Persson and Scaraggi [20] assume Gaussian roughness but have questioned the validity of Fuller and Tabor's theory, because they make the asperity approximation, and therefore they neglect interaction effects and the possibility of reaching full contact. These papers clearly show the complexity of the problem which has so far received no clear general understanding, and we shall attempt here a model which, despite the initial strong assumption of neglecting elastic deformations, will give some simple results, given its simplicity. In fact, we do not need to make assumption over the type of height distribution, and we can therefore concentrate on the effect of different distributions on the decay of adhesion with roughness amplitude.

In the general case of a rough surface described by unbounded distributions of heights, it would seem in principle that, the profile being rigid, it would be pushed away from the repulsive component of the Lennard–Jones forces. However, this is a limit condition and one should consider that in any finite realization of a surface, even if in principle fitting an unbounded statistical distribution, there is a highest peak. Further, non-Gaussian profiles have long been recognized to be important, already form the very milestone contribution of Greenwood and Williamson [11]. Indeed, that paper in Fig. 6 shows a surface of mild steel which had been abraded and then slid against copper, resembling a truncated Gaussian as in the process later on studied by Borucki [3] and Borucki et al. [4] for Chemical Mechanical Polishing, based on Archard wear law. Man-made surfaces, "Structured", "Textured" or "Engineered" Surfaces ([9], for example) today exist with many

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different functions, that call for theories different from Gaussian. Limiting the attention to pure mechanical contact, exceptions to Gaussian models are McCool [14] who suggested two parameter Weibull distributions for asperity heights, noticing that it is capable of accounting for skewness, either positive or negative, in the asperity height distribution. Also, Chilamakuri and Bhushan [6], Kotwal and Bhushan [13] develop some non-Gaussian models of contact, and Adler and Firman [1] show metallic surfaces, including abraded stainless steel as well as previous data, indicating the need for such a non-Gaussian model.

In order to look at the problem with incremental complexity, we start from the case of a Weibull distribution, moving then to a self-affine Gaussian distribution, and to a truncated Gaussian surface originated from a wear process.

2. Formulation

Since we are going to neglect elastic deformations, we have to assume that there cannot be interpenetration of the rough (rigid) body with the rigid wall. Therefore, we are going to consider only roughness which is described either by an intrinsically bounded distribution (such as Weibull), or by finite discrete realization of nominally unbounded distributions (such as Gaussian). We assume that the rigid wall is $h=d$ and that we cannot interpenetrate the halfplane $h<d$, the rough body will be occupying some areas of the right halfplane $h>d$, with a distribution $\phi(h)$ defined between $h_1<h$, where $h_1>d$, and we can in general write the traction

$$t=\int_{h_1}^{\infty}\phi(h)p_{LJ}(h-d)\,dh \tag{1}$$

We follow the standard definition of LJ as (positive p_{LJ} if attractive)

$$p_{LJ}(h)=B\left[\left(\frac{d_c}{h}\right)^n-\left(\frac{d_c}{h}\right)^m\right] \tag{2}$$

where usually $n=3$ and $m=9$. The constant d_c is introduced as a cut-off length which corresponds to the interaction potential characteristic distance. A much used alternative purely adhesive potential is [19]

$$p_a^+=B^+\left(\frac{d_c}{h+d_c}\right)^n \tag{3}$$

which is convenient when the repulsive part of the LJ potential is taken by classical Signorini boundary condition of contact at zero separation.

The constant B is chosen in order to make in any case the integral of the adhesive part equal to $\Delta\gamma$

$$\int_{d_c}^{\infty}p_{LJ}(h)\,dh=\int_{0^+}^{\infty}p_a^+(h)\,dh=\Delta\gamma \tag{4}$$

The peak value of p_{LJ} is at $h/d_c=1.2$ and in this point we define theoretical strength, so that $p_{LJ}=0.385B=\sigma_{th}$ and $B=\sigma_{th}/0.385$, whereas the peak value for p_a^+ is obviously at 0. Hence, in order to have the same $\Delta\gamma$, the peak tensile strength is double than the standard LJ potential, and hence

$$p_a^+=B^+=2\left(p_{LJ}\right)_{\max}=2\sigma_{th}.$$

In comparing the results using the two models (either full LJ and equilibrium between repulsion and attraction), or purely adhesive LJ, and mechanical contact at the highest peak/summit of the surface, we shall consider this discrepant factor.

In a DMT model of a rough contact [19], it is clear that if $P(h)$ is the distribution of interfacial separations due to compressive stresses alone, then the adhesive contribution to the force is obtained from (1) for $d=h_1=0$

$$p_{ad}=\int_{0^+}^{\infty}P(h)p_a(h)\,dh \tag{5}$$

whereas the compressive contribution p_N should be computed with a theory of pure mechanical contact, although the distribution $P(h)$ is not simple to obtain. DMT theory neglects elastic deformations due to the adhesion terms, and only considers deformations for the mechanical compressive solution. Here, we further neglect also these deformations (and hence we replace $P(h)$ by the original $\phi(h)$), which is realistic if an equivalent of the Tabor parameter is small. For the sphere this implies [21]

$$\mu=\left(\frac{R\Delta\gamma^2}{E^{*2}}\right)^{1/3}/d_c<<1 \tag{6}$$

which in turn means obviously large elastic modulus with respect to the theoretical strength, and "small" sphere radii with respect to the interaction potential distance. For the sinusoid, Wu [22] defined a modified Tabor parameter for a single sinusoid. However, but for a rough surface, we cannot identify a single parameter, although we probably imply not too large roughness wavelengths and hence amplitudes.

With a truncation in the height distribution, the simplified form of the LJ potential permits to say that pull-off will occur when the distribution of heights is located exactly at the zero-level point of the LJ distribution (Fig. 1).

3. Results for Weibull distributions

There are endless distributions of bounded tail on one end. In this paragraph, we shall start by considering the 2-parameters Weibull distribution with scale parameter $b>0$ and shape parameter $a>0$, which has been already suggested for surfaces with skewness by McCool [14]. Weibull's PDF is defined for $h>0$ in dimensionless coordinates as $W(h;a,b)=ab^{-a}h^{a-1}e^{-h^a b^{-a}}$. The tensile stress at pull-off is therefore found from (1) when the distribution is just in contact at zero separation

$$p_{pull-off}(a,b)=\int_0^{\infty}W(h;a,b)p_a^+(h)\,dh \tag{7}$$

Some example results are plotted in Fig. 2, for $a=1,2,3$ together with some inverse power law asymptotic limit curves. On the x-axis, we have used the root mean square value of the roughness, in order to have an easy comparison, and the relationship with the scale parameter is well known to be

$$h_{rms}=b\sqrt{\Gamma\left(1+\frac{2}{a}\right)-\Gamma\left(1+\frac{1}{a}\right)^2} \tag{8}$$

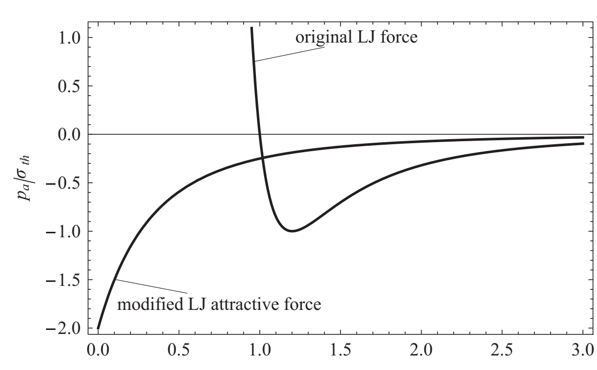

Fig. 1. The Lennard–Jones potential $p_a(h)/\sigma_{th}$ (labelled as "original LJ force") and the "modified LJ attractive force" $p_a^+(h)$ as a function of the distance h/d_c.

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