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The influence of geometrical and rheological non-linearity on the calculation of rubber friction

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ABSTRACT

We discuss the influence of geometrical and rheological non-linearities on the prediction of rubber friction and true contact area for rough sliding interactions. In particular, we compare the results of a linearly-viscoelastic linear-contact model, formulated in the Fourier space, with those obtained from non-linear finite element calculations. A sinusoidal rigid profile indenting a rubber block is here considered for simplicity, whereas the effects of non-linearity are evaluated by varying the aspect ratio, loading conditions and sliding speed of the contact interface. It is found that accurate friction predictions can be obtained through the linear viscoelastic model, provided that the roughness under investigation features moderate values of root mean square slopes, whereas non-linear finite element computations should be adopted for large root mean square slopes.

1. Introduction

In the last few decades, friction, wear and adhesion of polymers have been the subject of intense theoretical [1–23] and experimental research [1,2,24,25,3,4,26,15,21,27–31], motivated by a significant number of applications ranging from classical machine elements to bio-tribological contacts. The mechanisms originating the macroscopically observed behavior are nowadays quite clearly understood, at least qualitatively.[1] However, the quantitative prediction of these phenomena is still an open issue, and this can be ascribed to the complexity of the molecular-to-macro-phenomena involved in the contact dynamics.

Herein we focus in particular on rubber friction, which is well known to be hysteretic and multiscale in nature [6]. A representative contact geometry consists in a rubber block sliding on a rough rigid surface. This simple problem is relevant to a number of applications including dynamic seals, tire tread-road contact, medical devices (e.g. gliding devices) and bio-tribological interfaces. In this representative case, assuming a steady-sliding steady-worn rough contact [8] and dry conditions, rubber friction involves two main micro-mechanisms of dissipation, i.e.

$$\mu = \mu_r + \mu_{ad}, \tag{1}$$

with μ_r as the micro-rolling friction and μ_{ad} as the contribution of the true shear stresses acting in the area of real contact. The meaning of these two terms will be expanded upon in the following. In the case of wet contacts, other terms originating from the fluid viscous dissipation have to be added to the dry contribution, see e.g. the recent discussion in Ref. [23].

The term micro-rolling friction for μ_r indicates that the dissipation mechanism is shared with the more classical rubber rolling friction of e.g. a rigid ball rolling on a rubber block. This contribution originates from the pulsating deformation resulting from the indentation of the rough rigid profile sliding over the rubber bulk. The second mechanism of dissipation μ_{ad} is instead related to the shear stresses acting in the area of real contact, and

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[1] We note that even the fundamental chemo-mechanical modeling of bulk complex polymeric networks is a task whose complexity requires a multi-disciplinary approach. As an example, in the (relatively) simple case of a filled rubber compound (say a filled SBR), the rheological properties exhibit non-linearity related to filler-filler interactions, filler-rubber interactions, degree of cross-linking, functionalization of the filler, localized slip of bindings, reduction of the entropic states, to cite just a few. These phenomena are widely accepted to provide a physical justification to the origin of some rubber rheological behaviors, known under the name of e.g. Payne effect, Mullins effect, etc. Despite the fundamental understanding of such non-linear phenomena has reached a qualitative level from several years, their best quantitative prediction is mainly based on phenomenological or fitting models. Furthermore, when extending the consideration of such non-linear phenomena to the realm of friction and wear, where multiple length scales (up to the macro-scale, including interfacial phenomena such as the Schallamach waves, to cite one) are added to the length scale regulating the rheological processes described above, the complexity of the problem can only be handled, at the moment, with models (analytical or numerical) which are intrinsically qualitative.

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Nomenclature

General variable

$\mathbf{x}$	Generic position with components (x, y) in the reference frame moving with the rigid body
$\mathbf{q}$	Wave vector, $\mathbf{q} = \left(q_x, q_y\right)$
t	time
$\mathbf{v_0}$	Sliding velocity
v^*	Characteristic velocity, $v^* = L_0/(2\pi\tau)$
A_c	Contact area
p_0^*	Full contact pressure in the rubbery regime
D_h	Deborah number, $D_h = v_0\tau_m/L_0$

VHS model variables

$u(\mathbf{x})$	Separation field
$t_N(\mathbf{x})$	Contact pressure in the real domain

FE model variables

$\mathbf{F}_e$	Elastic part of the deformation gradient
$\mathbf{F}_e^k$	Elastic part of the deformation gradient for the $k-th$ Maxwell element
$\mathbf{F}_v$	Viscous part of the deformation gradient
$\mathbf{F}_v^k$	Viscous part of the deformation gradient for the $k-th$ Maxwell element
$\mathbf{C}_v$	Right Cauchy-Green tensor for the viscous part
$\mathbf{\Pi}$	Strain energy function for neo-Hooekan material
$\boldsymbol{\sigma}$	Cauchy stress tensor
$\boldsymbol{\sigma}_{eq}$	Equilibrium part of the Cauchy stress tensor
$\boldsymbol{\sigma}_{eq}^k$	Non-Equilibrium part of the Cauchy stress tensor function of the $k-th$ Maxwell element
$\overline{p}$	Average contact pressure
$\mathbf{g}_N$	Gap vector
$\overline{\mu}$	Macroscopic friction coefficient
$\overline{\mu}_{av}$	Time averaged macroscopic friction coefficient
t_i, t_f	Initial and final instants of the time averaging period, respectively
$\mathbf{N}$	Normal unit vector
L_{ij}	Residual for each point of the grid of coordinates i,j
C_{ij}^{hk}	Compliance matrix
ϵ_L, ϵ_u	Tolerances
$w_z(\mathbf{x})$	Out-of-contact plane displacement field
$\overline{u}$	Average separation
$h(\mathbf{x})$	Roughness surface, with $\langle h(\mathbf{x})\rangle = 0$
δ	Contact penetration
F_N	Normal load
F_T	Friction force
μ_r	Micro-rolling friction
$t_N(\mathbf{q})$	Contact pressure in the Fourier domain

Rubber characteristics

ν	Poisson's ratio
$E_{r\infty}$	Reduced low frequency rubber elastic modulus (rubbery regime), $E_{r0} = E(0)/(1-\nu^2)$
Λ, μ	Lamé constants, respectively $\Lambda = E\nu/[(1+\nu)(1-2\nu)]$ and $\mu = E/[2(1+\nu)]$
$E(\omega)$	Rubber complex viscoelastic modulus
$E_r(\omega)$	Rubber reduced complex viscoelastic modulus, $E_r(\omega) = E(\omega)/\left(1-\nu^2\right)$
E_k	$k-th$ term of the rubber relaxation spectrum
τ_k	$k-th$ rubber relaxation time
τ_m	Rubber relaxation time corresponding to the maximum loss tangent
E_∞	High frequency rubber elastic modulus (glassy regime)
$E_{r\infty}$	Reduced high frequency rubber elastic modulus (glassy regime), $E_{r\infty} = E(\infty)/(1-\nu^2)$
E_0	Low frequency rubber elastic modulus (rubbery regime)

Roughness characteristics

L_0	Wavelength of the Westergaard profile
Δ	Amplitude of the Westergaard profile
q_0	Spatial frequency of the Westergaard profile, $2\pi/L_0$
m_2	Mean square slope

strictly depends on the physics of interface bonding/debonding [32]. In particular, the simplest picture for sliding contact of a polymer (e.g. as occurring in the case of smooth and clean rubber in sliding contact with a smooth glass substrate) has been discussed by Shallamach [32,6], and involves the bonding/debonding process of polymer chains from the substrate depending on the sliding velocity. The corresponding dissipation is related to the release of phonons propagating in the bulk of the solids.

Additional factors such as adhesion [33,9,15,34], contamination, wear-dependent rubber layering and roughening [35], tribocharging [36], flash temperature effects [11,37] and several other interface phenomena [38] are well known to substantially alter the two previous frictional mechanisms, thus making the overall friction quite complex to capture experimentally (in each of its contributions) as well as to predict theoretically in quantitative terms. We stress that despite a substantial improvement in the fundamental understanding of rubber friction, the reliable quantitative prediction of friction still remains an open issue.

On the analytical side, rubber friction is usually calculated in the framework of (i) linear viscoelasticity with infinitesimal deformations [8–11,38,39,14,16,20–23,40], as well as recurring to (ii) the Reynolds roughness assumption, i.e. the roughness square slope $\left\langle \nabla h^2 \right\rangle \lll 1$, where h is the surface roughness, with $\langle h \rangle = 0$ (i.e., the roughness is computed with respect to a reference mid-plane, see Fig. 1(a)). The above assumptions allow for the use of the well-known viscoelastic half-space (VHS) theory [41] in the modeling of the deformation response of generic contacting surfaces. This approach is typically adopted in multiscale [8] as well as in multi-asperity [42,39] viscoelastic contact mechanics theories, and also in boundary element numerical formulations [43,20,22,23,40]. On the other hand, finite element (FE) approaches are able to remove both assumptions and provide a prediction of rubber friction in the finite deformation framework, for arbitrary geometry of the contacting bodies and arbitrary constitutive behavior of the material, albeit with a much higher computational cost. However, to the best of our knowledge, how these assumptions quantitatively affect the rubber friction calculations has never been investigated before.

The multiscale nature of the micro-rolling frictional contribution μ_r for contact to rough surfaces is reflected e.g. in the well-known analytical theory by Persson [8]. At a contact scale of representative size $\lambda = 2\pi/q$, where $\zeta = q/q_0$ ($q = |\mathbf{q}|$, with $\mathbf{q}$ as the