ARTICLE IN PRESS

Tribology International ▪ (▪▪▪▪) ▪▪▪–▪▪▪

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

Adhesion of an axisymmetric elastic body: Ranges of validity of monomial approximations and a transition model

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ARTICLE INFO

Article history:
Received 25 August 2015
Received in revised form
3 November 2015
Accepted 10 February 2016

Keywords:
Adhesion: Contact, Forces
Contact: Elastic
Model: Analytical

ABSTRACT

The Johnson–Kendall–Roberts and the Maugis models have been used to model the adhesion of spherical elastic bodies (represented by paraboloids). Recent work has investigated adhesion of higher-order monomial shapes. These results differ significantly from those of paraboloids. Given that any practical shape will not be "exactly" either a paraboloid or a higher-order monomial, the question arises as to the ranges of validity of these models and what is the requirement, if any, for a transition model. In this investigation the ranges of validity of these models are established by considering a two-term transition model. Furthermore the need for using this transition model is shown to depend not only on the geometry of the bodies but also on material properties.

1. Introduction

The elastic contact with adhesion of spherical elastic bodies (whose surface profiles are approximated by a single second-order term) was investigated by Johnson et al. (JKR model,) [1] and Derjaguin et al. (DMT model) [2]. Both models add to the Hertz contact model the effect of the Dupré energy of adhesion w, which is defined as the work per unit area needed for the reversible, isothermal separation of two solids. This quantity, also known as the work of adhesion, is given by $w=\gamma_1+\gamma_2-\gamma_{12}$, where γ_1 and γ_2 are the surface energies of the two bodies in contact and γ_{12} is the interfacial energy. If the bodies are identical then, $w=2\gamma$. The JKR model adds the adhesion effect by minimizing the total potential energy which includes the Dupré energy of adhesion, giving a pull-off force of $\frac{3}{2}\pi wR$. The DMT model adds the adhesive stresses outside the contact region while maintaining the Hertz stress distribution inside the contact area, thereby obtaining a pull-off force of $2\pi wR$.

Using the Dugdale model from fracture mechanics, Maugis introduced a model (also known as the M-D model) [3] which demonstrates a continuous transition between the JKR and DMT theories based on a parameter λ that is closely related to the Tabor parameter μ [4]. The Tabor parameter is a measure of the ratio of the elastic deformation to the range of surface forces. The Maugis parameter can be expressed in terms of Tabor parameter as $\lambda=1.16\mu$. Maugis showed that as $\lambda\to0$ the DMT model is applicable whereas when $\lambda\to\infty$ the JKR model is called for. For practical purposes λ less than about 0.1 is DMT and λ greater than about 3 is JKR.

Although the three models mentioned above assume that the bodies in contact are spherical, their surface geometries are in fact approximated by a single second-order term. In their research on the friction force between an atomic force microscope (AFM) tip and a nominally flat surface, Carpick et al. introduced an extended JKR model applicable to axisymmetric elastic bodies in contact with a surface profile described by a single n-th order term, i.e. Cr^n [5,6]. Later, Zheng et al. developed an analytical model to extend the M-D theory to asperities with such power-law geometries, called the M-D-n model [7]. Grierson et al. then performed a finite element analysis and experimental measurements which agreed well with the analytical model [8].

Some questions naturally arise in deciding whether to use a second- or higher-order approximation of a surface profile [10]. For example, if the real surface profile is "exactly" either a 2nd-order or 4th-order shape then it is clear which shape to use in calculating the pull-off force, the force vs. the contact area, and the force vs. the penetration. However, a realistic shape could no doubt be approximated to different degrees of accuracy by either a 2nd-order or a 4th-order shape. The choice of which approximation to use is expected to depend upon how close the actual profile is to each of these shapes, but does it also depend upon a parameter involving the material properties? Furthermore, under what conditions does the shape need to be described by more than one term, *i.e.* a combination of a 2nd-order term and a 4th-order order term? Another issue is the choice of using the JKR and its extended

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http://dx.doi.org/10.1016/j.triboint.2016.02.012
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model as opposed to the more complicated Maugis and its extended model. For a pure 2nd-order surface profile these regimes are reasonably clear. However do these regimes differ for a two-term approximation? By extending JKR and M-D mo`dels to a surface profile with two terms, we hope to answer these questions.

2. JKR model extension for two terms

2.1. Problem formulation

If a pressure distribution is applied to a circular region of an elastic half space of radius a, a closed-form solution for the normal component of surface displacement can be found. Consider a cylindrical coordinate system (r, θ, z) and apply on $z=0$ the pressure distribution given by

$$p(r) = p_m\left(1-\frac{r^2}{a^2}\right)^n \qquad r<a \tag{1}$$

where the maximum contact pressure is p_m, which occurs at $r=0$.

Following the procedure of Johnson [9] and referring to Fig. 1, the displacement of the surface at point B can be found using a local polar coordinate system (s, ϕ) with origin at point B. At a distance s from B, the pressure $p(s,\phi)$ that acts on a small element of area corresponds to a force with magnitude $p(s,\phi)sdsd\phi$. The displacement at B resulting from the pressure distributed on the whole area is then

$$u_z = \frac{1-\nu^2}{\pi E}\iint_A p(s,\phi)dsd\phi \tag{2}$$

where ν is Poisson's ratio and E is Young's modulus.

The resultant force P is obtained by integration of the pressure over the circular region

$$P = \int_0^a 2\pi r p(r)dr \tag{3}$$

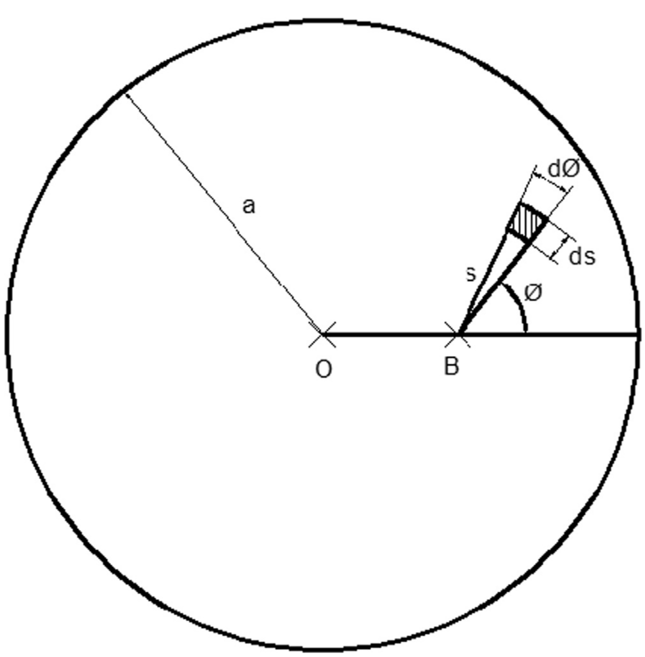

Fig. 1. Schematic of axisymmetric elastic body with center O showing point B, at distance r from the center, used to calculate vertical displacement u_z.

The value of the exponent n in the above pressure distribution depends on the surface profile of the elastic bodies in contact. For example $n=\frac{1}{2}$ represents the pressure distribution between two spherical elastic solids in frictionless contact without adhesion as obtained by Hertz (1882), with a maximum contact stress p_1 and can be written in terms of the resultant force P_1 as

$$p(r) = \frac{3P_1}{2\pi a^2}\left(1-\frac{r^2}{a^2}\right)^{\frac{1}{2}} \qquad r<a \tag{4}$$

The displacement expressed in terms of the resultant force P_1 is

$$u_z(r,0) = \frac{3P_1}{8E^*a}\left(2-\frac{r^2}{a^2}\right) \qquad 0<r<a \tag{5}$$

$$u_z(r,0) = \frac{3P_1}{4\pi a E^*}\left[\left(\left(2-\frac{r^2}{a^2}\right)\sin^{-1}\frac{a}{r}\right)+\left(\sqrt{\frac{r^2}{a^2}-1}\right)\right] \qquad r>a \tag{6}$$

where E^* is the composite Young's modulus defined by

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2} \tag{7}$$

where E_1 and E_2 are the elastic Young's moduli of elastic bodies 1 and 2 respectively, and ν_1 and ν_2 are Poisson's ratios of bodies 1 and 2 respectively.

A pressure distribution in which $n=-\frac{1}{2}$ results in a uniform normal displacement of the circular region $(r<a)$ and corresponds to the pressure, p_2, exerted by a flat-ended, frictionless, rigid punch pressed against an elastic half space with contact radius a [9]. In this case, the displacement at the surface can be expressed in terms of the resultant force P_2 as

$$u_z(r,0) = \frac{P_2}{2E^*a} \qquad 0<r<a \tag{8}$$

$$u_z(r,0) = \frac{P_2}{\pi a E^*}\sin^{-1}\frac{a}{r} \qquad r>a \tag{9}$$

By using the fracture mechanics concept of the stress intensity factor K_I at the edge of the circular region, the force needed to separate the punch from the half-space may be obtained as a function of the Dupré energy of adhesion w

$$K_I = \lim_{r\to a} p(r)\sqrt{2\pi(a-r)} \tag{10}$$

By setting K_I equal to its critical value

$$-\frac{P_2}{2\sqrt{\pi a^3}} = \sqrt{2wE^*} \tag{11}$$

is obtained. The displacement inside the circular region is therefore:

$$u_z(r,0) = -\sqrt{\frac{2\pi w a}{E^*}} \qquad 0<r<a \tag{12}$$

If the exponent value$n$ of the pressure distribution is equal to $\frac{3}{2}$, with a maximum contact pressure p_3 and a resultant force P_3, the corresponding displacements are

$$u_z(r,0) = \frac{15P_3}{16aE^*}\left(1-\frac{r^2}{a^2}+\frac{3}{8}\frac{r^4}{a^4}\right) \qquad 0<r<a \tag{13}$$

and

$$u_z(r,0) = \frac{15P_3}{8\pi a E^*}\left[\left(\left(1-\frac{r^2}{a^2}+\frac{3}{8}\frac{r^4}{a^4}\right)\sin^{-1}\frac{a}{r}\right)+\left(\sqrt{\frac{r^2}{a^2}-1}\right)\left(\frac{3}{4}-\frac{3}{8}\frac{r^2}{a^2}\right)\right], \quad r>a \tag{14}$$

A superposition of these three pressure distributions results in the following surface displacements

for $0<r<a$

Please cite this article as: Oweiss F, Adams GG. Adhesion of an axisymmetric elastic body: Ranges of validity of monomial approximations and a transition model. Tribology International (2016), http://dx.doi.org/10.1016/j.triboint.2016.02.012