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A smoothed particle hydrodynamics (SPH) model for simulating surface erosion by impacts of foreign particles

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A B S T R A C T

This paper presents a smoothed particle hydrodynamics modeling procedure to simulate surface erosion by impacts. The present SPH is improved based on the standard SPH formulation, by developing SPH formulations for the constitutive relations to describe plastic behavior and ductile fracture process. The improved SPH method is first applied to simulate impacts of particles on ductile targets. Two modified schemes in terms of density correction and kernel gradient correction are adopted to improve the accuracy of the SPH approximation. The present SPH model is applied to simulate the erosion process by angular particles. The results are compared with available experimental data, and good agreements have been achieved in terms of the crater profile and particle kinematic results.

1. Introduction

The material removal caused by impacts of particles is one of the most common surface erosion process in many engineering systems. Impact on ductile materials using foreign particles may be viewed as either constructive useful engineering technique (e.g. shot blasting [1], abrasive jet [2]) or destructive harmful processes (e.g. impeller erosion [3], pipe erosion [4,5]). Study of the mechanisms of surface erosion by impacts is of critical importance in the advancements of engineering technique or in reducing possible erosive wears in engineering systems.

Material deformation and removal are two major concerns involved in surface erosion by impacts. For ductile materials, the impacts of the foreign particles result in localized plastic strain [6,9] on the surface of the contact site and material is removed when the strain exceed a threshold value [7]. It has been known that material removal does not necessarily occur during the process of foreign particles impacting on ductile targets. It depends on many factors, some of which may individually or synthetically determine the erosion mechanisms, such as particle velocity, angle of attack, particle shape, size of particle, etc. Usually, correlations between erosion rate and erosive factors are obtained through experiments by measuring mass loss or analyzing eroded surface. However, the interaction of these factors makes it difficult to examine the mechanisms experimentally. For example, it is hard to observe the dynamic process of material removal (or material spallation) and to analyze the dependency on each erosion factor through experiment, due to the complexity of the process occuring in a very short time. Computer modeling allows effective and detailed studies to investigate all these erosion factors separately. It can serve as a valuable complement to the experiment, helping to reveal the fundamental behaviors involved in the erosion process and predict the performace of the system with respect to different erosion factors.

The Finite element method (FEM) is an effective numerical method in solving problems in solid mechanics and has been applied widely to model the surface erosion impacted by spherical particles [7,11–17]. With appropriate constitutive material models, FEM is capable to simulate the relevant damage phenomena in surface erosion process. These models can be validated by experimental observations or analytical solutions for simple benchmark problems. However, these FEM models mainly focused on predictions of erosion rate quantitatively or analysis of erosion mechanisms qualitatively. It is difficult to observe and reveal the erosion mechanisms due to the complex dynamic process of material fragmentation. Moreover, actual foreign particles usually have complex geometry shape with angularity. Impacts of angular particle can cause large plastic deformation and rapid material removal, which may result in the heavily distorted elements leading to poor accuracy. Therefore, standard FEM may not be the suitable method for modeling surface erosion involving large plastic deformation and material removal. It is known that most the element-based numerical methods have the similar difficulties to handle large deformation and material fragmentations.

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Smoothed particle hydrodynamics (SPH) is a Lagrangian meshfree particle method that does not use elements. It was initially developed for astrophysical problems [18–20]. Since its invention, SPH has been extensively applied in the many fields of science and engineering including fluid mechanics and solid mechanics, such as free surface flows [21,22], viscous flow [23,24], high velocity impacts [25–27], geophysical flows [28,30], etc. As a meshfree method, SPH uses particles to represent the materials and to carry the field variables such as density, stress, and to approximate the governing equations. These particles have a spatial distance (named as the "smoothing length"), over which their properties are "smoothed" by a kernel function. SPH has great advantages over the element-based numerical methods to deal with large deformation and material removal due to its particle nature. Therefore, SPH method may be a better option for the simulation of surface erosion by impacts.

In the past few years, several preliminary applications of SPH method to surface erosion by impacts have been performed and some encouraging results have been obtained. For example, Wang and Yang [31] investigated multiple impacts of spherical particles on Ti–6AL–4V using the SPH method. The predicted erosion dependency on impact factors agrees well with the analytical and experimental results. Takaffoli [29] proposed a SPH model to simulate the impact of single angular particles on *AL6061-T6* targets. The dynamic process of material removal caused by impacts was first simulated by the numerical method and the resluts showed that SPH method can account for both material deformation and chip separation. It demonstrated that the SPH method is able to capture the major fundamental dynamic behavior of surface erosion by impacts. However, the traditional SPH method encounters the problem of low accuracy. The numerical simulations of surface erosion using traditional SPH are limited and more detailed studies are required to fully unleash the fullest potential of SPH techniques.

This paper is to establish a general SPH framework for modeling surface erosion by impacts which comprises the reproduction of material behavior in terms of both plastic deformation and material removal, and improvement of numerical accuracy. In Section 2, the general concepts of the SPH modeling for continuum material are given, and the standard SPH formulations are presented. Here the low accuracy of the conventional SPH algorithm is discussed. The improved SPH method is then introduced and described in details. Two modified schemes for density correction and kernel gradient correction are implemented. In Section 3, the numerical model is built based on the rigid-plastic theory and a particle contact algorithm is implemented for calculating the interaction force between rigid body and targeted material. The effectiveness of presented SPH model is validated in Section 4 by numerical examples including square particle erosion on *AL6061-T6* and rhombic particle erosion on *OFHC copper*.

2. SPH modeling

2.1. Governing equations and SPH formulations

The governing equations of ductile targeted material which consist of mass and momentum conservation equations can be expressed following:

$$\frac{D\rho}{Dt} = -\rho\frac{\partial v^{\alpha}}{\partial x^{\alpha}} \tag{1}$$

$$\frac{Dv^{\alpha}}{Dt} = \frac{1}{\rho}\frac{\partial\sigma^{\alpha\beta}}{\partial x^{\beta}} + f^{\alpha} \tag{2}$$

where α and β denote the Cartesian components x, y with the Einstein convention applied to repeated indices; ρ is the material density; t is the time; v is the velocity; $\sigma^{\alpha\beta}$ stands for the total stress tensor; the total stress tensor $\sigma^{\alpha\beta}$ has two parts, one is isotropic pressure p and the other one is deviatoric shear stress $\tau^{\alpha\beta}$;f^{α} is the component of acceleration caused by external force.

In SPH framwork, the computation domain is discretized by a finite number of particles that carry individual mass and occupy individual space. The governing equations of continuum mechanics are then discretized basing on space distribution of these particles. Since the computation domain has been discretized with particles, the field function at a particle can be obtained simply through summations over all particles within the support domain of the particle using a kernel function, which is the so-called particle approximation process. According to the continuity equation (Eq. (1)) and momentum equation (Eq. (2)), the governing equations can be expressed as [32]

$$\begin{cases} \frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} v_{ij}^{\beta} \cdot \frac{\partial W_{ij}}{\partial x_i^{\beta}} \\ \frac{dv_i^{\alpha}}{dt} = \sum_{j=1}^{N} m_j \left[\frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i \rho_j} - \Pi_{ij}\delta^{\alpha\beta} \right] \cdot \frac{\partial W_{ij}}{\partial x_i^{\beta}} + f_i^{\alpha} \\ \frac{de_i}{dt} = \frac{1}{2}\sum_{j=1}^{N} m_j \left(\frac{P_i + P_j}{\rho_i \rho_j} + \Pi_{ij} \right) v_{ij}^{\beta} \cdot \frac{\partial W_{ij}}{\partial x_i^{\beta}} + \frac{\tau_i^{\alpha\beta}\epsilon_i^{\alpha\beta}}{\rho_i} \\ \frac{dx_i^{\alpha}}{dt} = v_i^{\alpha} \end{cases} \tag{3}$$

where W_{ij} is the smoothing function and expressed by $W_{ij} = W(|\boldsymbol{x_i} - \boldsymbol{x_j}|, h)$,and $\nabla_i W_{ij}$ is the gradient of kernel, $\nabla_i W_{ij} = \frac{\boldsymbol{x_i} - \boldsymbol{x_j}}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} = \frac{\boldsymbol{x_{ij}}}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}}$. The term (Π_{ij}) is called artificial viscosity and is used to reduce the unphysical oscillations in the numerical results around the shocked region. Of several proposals for artificial viscosity developed so far, the most widely applied is derived by Monaghan [20]

$$\Pi_{ij} = \begin{cases} \frac{-\alpha\bar{c}_{ij}\mu_{ij} + \beta\mu_{ij}^2}{\bar{\rho}_{ij}} & \vec{v}_{ij} \cdot \vec{x}_{ij} < 0 \\ 0 & \vec{v}_{ij} \cdot \vec{x}_{ij} \geq 0 \end{cases} \tag{4}$$

where $\mu_{ij} = \frac{h_{ij}\left(\vec{v}_{ij} \cdot \vec{x}_{ij}\right)}{\left|\vec{x}_{ij}\right|^2 + 0.01h_{ij}^2}$, $\bar{c}_{ij} = (c_i + c_j)/2$, $\bar{\rho}_{ij} = \left(\rho_i + \rho_j\right)/2$, $h_{ij} = (h_i + h_j)/2$, c is the speed of sound; α, $\beta$are constantsandshould bechosen according to particular applications.

2.2. Improved algorithm

As we discussed previously, the standard SPH method is robust but usually has low accuracy as it cannot exactly reproduce quadratic and linear functions. Also, the accuracy of the method is closely related to the distribution of particles. During the past decade, many different attempts have been made and approaches have been proposed to improve the accuracy of SPH approximation [33–41]. In this paper, two modified schemes in terms of density correction and kernel gradient correction are adopted, which have been proved effectively to improve computational accuracy [22,42].

For the density correction, Moving Least Squares (MLS) approach [38], which is a interpolation scheme on irregularly scattered points, is used to correct the density field periodically. This strategy had been applied successfully by Colagrossi and Landrini [42] in SPH simulation of interfacial flows. And it shown that the linear variation of the density field can be exactly reproduced using this correction scheme. Besides, it was also found that a smoother pressure field was obtained, which may be helpful in filtering the spurious pressure oscillation in this simulation.