Tribology International 94 (2016) 458–463

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

A novel Reynolds equation of non-Newtonian fluid for lubrication simulation

Qianqian Yang *, Ping Huang, Yanfei Fang

School of Mechanical and Automotive Engineering, South China University of Technology, Guangzhou 510640, China

ARTICLE INFO

Article history:
Received 23 June 2015
Received in revised form
27 August 2015
Accepted 11 October 2015
Available online 17 October 2015

Keywords:
Reynolds equation
Non-Newtonian
Lubrication
Simulation

ABSTRACT

The non-Newtonian phenomenon is significant in hydrodynamic lubrication of some special lubricants, or elastohydrodynamic lubrication (EHL). However, the conventional methods to solve the non-Newtonian lubrication problem are either too complicated or inaccurate. This paper puts forward a Reynolds equation for general lubrication problem of the non-Newtonian fluid by treating the lubricant flow as the superposition of the Poiseuille flow and Couette flow. Then, as examples, a set of simulations for EHL in the line contact are presented to investigate the feasibility of the method. Finally, comparisons with the conventional methods establish the validity and simplicity of the method.

1. Introduction

After the disc machine has been used to study the elastohydrodynamic lubrication (EHL) since 1960s, it is found that the lubricant films were much weaker than that the theories predict from the Newtonian behavior [1]. It was realized that the lubricants behave in a highly non-Newtonian fashion when they pass through an EHL contact region [2]. The conditions in the EHL contact region are severe for extremely high pressure, usually above 0.5 GPa, and very high shear rate, typically 10^6–$10^8\ s^{-1}$ [3]. These conditions result in the lubricant experiencing a very large increase in the lubricant viscosity and a very high shear stress which produce the non-Newtonian behavior of the lubricant. The non-Newtonian behavior may exhibit shear-thinning/thickening, limiting shear stress, viscoelastic, or Maxwell behavior [4–6]. And some widely accepted constitutive equations have been proposed to describe the behavior [7–9].

With the development of the computer technology and numerical analysis, it is now possible to numerically simulate a variety of lubrication phenomena with different constitutive equations that have reasonable agreement with the experiments and the practice [10–12]. The main methods to solve the non-Newtonian lubrication problem are: (1) to obtain the Reynolds equation [13,14] by integrating their constitutive equations and other related equations directly [15], (2) to derive the Reynolds equation from the simplified Navier–Stokes equation and the continuity equations [16], and (3) to deduce the generalized Reynolds equation from a non-Newtonian constitutive equation [17].

The difficulty to theoretically obtain the modified Reynolds equation due to the complicated relationship between the shear stress and shear rate greatly limits the application of the first method. The Navier–Stokes equation is derived through the Cauchy equation and by specifying the stress tensor in terms of the viscosity and fluid velocity through the constitutive equation. The solution of it is flow velocities with the assumption of constant density and viscosity. The assumptions bring about significant errors to the velocity so that limit its application, not to mention the difficulty of calculation.

Therefore, the generalized Reynolds equation method maybe is the only suitable one to solve the general non-Newtonian lubrication problem. This method is based on the generalized Newtonian fluid model. And the generalized viscosity of it is simplified to a non-linear function of the shear rate $\dot{\gamma}$ or shear stress τ, $\eta = \tau/\dot{\gamma}$. Although it is applicable, it uses an average Newtonian constitutive equation by integrating the generalized viscosity across the film thickness instead of the real non-Newtonian one. Besides, the generalized Newtonian fluid model is useful for steady simple shear flows, while the time dependent effects, the elongational effects, and the normal stress differences are not considered [3]. And, this method is not valid to capture the mechanics of fluids in all situations [18,19].

Many other available methods for non-Newtonian fluid lubrication problem have also been raised, such as the Reynolds equation put forward by Najii et al. [20], the homotopy analysis method (HAM) [21] and etc, but the complication and simplifications during the derivation greatly limit their applications.

* Corresponding author. Tel.: +86 13450422217.
E-mail address: qqyangscut@163.com (Q. Yang).

http://dx.doi.org/10.1016/j.triboint.2015.10.011
0301-679X/

This paper presents a unified Reynolds equation of general non-Newtonian fluid. Similar to the Navier–Stokes method, the flow velocity is focused on. The main idea is to take the viscous flow as the sum of the Poiseuille flow and Couette flow. And based on their special characteristics, the velocity is obtained easily. Then, the Reynolds equation can be deduced so that the other quantities, such as the pressure, load, elastic deformation and the lubricant film thickness can be found as a usual Newtonian lubrication problem. Finally, comparisons with the conventional methods are carried out, which have established the validity of the present method.

2. Unified Reynolds equation

2.1. Velocity field equation

A general lubrication system in the Cartesian coordinate system shown in Fig. 1 is studied. The assumptions during the following derivation are: the inertial and body forces of fluids are negligible, the flow is laminar flow, the pressure of lubricant does not vary in the film thickness direction, and no sliding occurs at the top and bottom surfaces. The assumptions are valid for most fluid lubrication applications.

The relationship between the shear stress and shear rate is nonlinear for the non-Newtonian fluid, and the general form can be written as:

$$\tau_x = f\left(\tfrac{du}{dz}\right)$$
$$\tau_y = f\left(\frac{dv}{dz}\right) \tag{1}$$

where $f(\cdot)$ represents a non-linear function, τ is the shear stress, $\dot{\gamma}$ is the shear rate, and u and v are the velocities of the fluid in the x and y direction.

The x direction is taken as an example to illustrate the method. Based on the balanced force of arbitrary "control volume" in the x direction, the equilibrium equation is [22]

$$\frac{\partial \tau_x}{\partial z} = \frac{\partial p}{\partial x} \tag{2}$$

where p is the pressure.

Substitute Eq. (1) into Eq. (2) yields

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_x}{\partial z} = \frac{\partial}{\partial z}\left[f\left(\frac{du}{dz}\right)\right] \tag{3}$$

Integrate Eq. (3) twice with respect to z, the velocity is

$$u = \int_0^z f^{-1}\left(\frac{\partial p}{\partial x}z + C_1\right)dz = F\left(\frac{\partial p}{\partial x}z + C_1\right) + C_2 \tag{4}$$

where C_1 and C_2 are the constants of integration, $f^{-1}(\cdot)$ is the inverse function of $f(\cdot)$, $F(\cdot)$ is the integral function of $f^{-1}(\cdot)$.

It is obvious that the analytical expression of Eq. (4) is not easy to get, and the analytical form of C_1 and C_2 are even harder to obtain with nonzero boundary velocities. Bird [23] found that the solution of the Newtonian fluid can be seen to be a sum of the solutions of the two separate problems of the Poiseuille flow and Couette flow. Therefore, we try to use it to the non-Newtonian fluid, and treat the viscous flow as the sum of the Poiseuille flow and Couette flow. The results show great agreement with the conventional methods which have established the feasibility of this treatment.

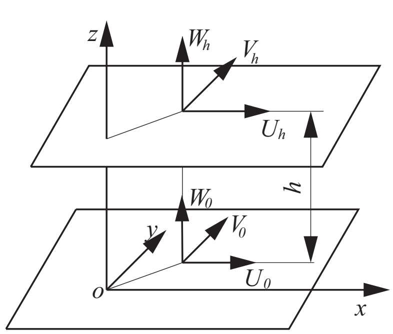

Fig. 1. General lubrication system.

2.1.1. Velocity of the Couette flow

The Couette flow is the wall driven laminar flow, and the velocity distribution of it is shown in Fig. 2. The Couette flow is driven by the viscous drag force acting on the fluid and the applied pressure gradient parallel to the flow direction. Since the pressure vertical to the flow direction has no effect on the Couette flow, then the equilibrium equation, Eq. (2), of it is

$$\frac{\partial \tau_x}{\partial z} = \frac{\partial p}{\partial x} = 0 \tag{5}$$

Therefore, the shear stress here is a constant. By using Eq. (4), Eq. (5) together with the boundary conditions of the velocity of the Couette flow, $u_C(0) = U_0, u_C(h) = U_h$, it gives:

$$u_C = \frac{U_h - U_0}{h}z + U_0 \tag{6}$$

2.1.2. Velocity of the Poiseuille flow

The Poiseuille flow is the pressure driven flow, and the velocity distribution of it is shown in Fig. 3. Unlike the Couette flow, it is significantly affected by the rheological properties of the fluid. And the two main features of the Poiseuille flow are:

(1) The velocity is symmetrical to the centerline, $z = h/2$, and $\frac{du_P}{dz}|_{z=h/2} = 0$.

(2) The boundary velocities are both zero, that is $u_p(0) = 0, u_p(h) = 0$.

According to feature (a) and Eq. (3)

$$0 = f\left(\frac{du_P}{dz}\right)_{z=h/2} = \frac{h}{2}\frac{\partial p}{\partial x} + C_1 \tag{7}$$

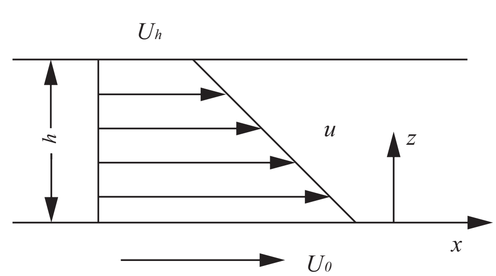

Fig. 2. Velocity distribution of the Couette flow.

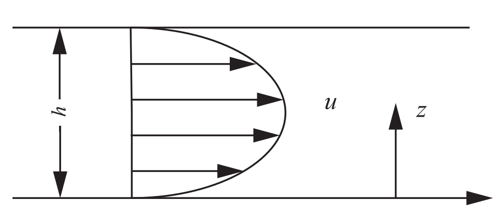

Fig. 3. Velocity distribution of the Poiseuille flow.