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Fault detection and isolation of LPV systems using set-valued observers: An application to a fixed-wing aircraft

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ABSTRACT

A novel fault detection and isolation (FDI) method using set-valued observers (SVOs), for uncertain linear parameter-varying (LPV) systems, is introduced. The proposed method relies on SVO-based model invalidation to discard models incompatible with measured data. When compared to the most common strategies in the literature, the suggested approach: (i) under suitable conditions, guarantees false alarms are avoided; (ii) unlike residual-based architectures, does not require the computation of thresholds to declare faults; (iii) has applicability to a wide class of plants. The performance of the proposed approach is assessed in simulation, using the full nonlinear model of a fixed-wing aircraft longitudinal dynamics.

1. Introduction

The field of fault detection and isolation (FDI) has been studied since the early 1970s Willsky (1976), and several techniques have, since then, been applied to different types of systems. An FDI device is key in several applications and, in particular, in those that are *safety critical*. Common examples of systems equipped with FDI devices include aircrafts and a wide range of industrial processes such as the ones described in the following references—Blanke, Izadi-Zamanabadi, Bogh, and Lunau (1997), Blanke, Staroswiecki, and Wu (2001), Isermann (1997), Patton and Chen (1997), Frank and Ding (1997), Esteban (2004), Collins and Tinglun (2001), Alwi and Edwards (2008), Longhi and Moteriù (2009), Mattone and De Luca (2006). For a survey of FDI methods in the literature, see, for instance, Hwang, Kim, Kim, and Eng Seah (2010). An FDI system must be able to bear with different types of faults in sensors and/or actuators, which can occur abruptly or slowly in time. Moreover, model uncertainty (such as unmodeled dynamics) and disturbances must never be interpreted as faults. Notwithstanding the hundreds (or maybe thousands!) of papers in the literature concerning this topic, there are still some open questions related to the performance guarantees provided by these devices.

An active deterministic model-based fault detection (FD) system (see Esteban, 2004, for a description of the typical FD classes available in the literature) is usually composed of two parts: a filter that generates residuals, which should be *large* under faulty environments; and a decision threshold, which is used to decide whether a fault is present or not—see Willsky (1976), Patton and Chen (1997), Esteban (2004), Frank and Ding (1994), Massoumnia (1986b), Besançon (2003), Bokor and Balas (2004), Blanke, Kinnaert, Lunze, Staroswiecki, and Schröder (2006), Puig, Quevedo, Escobet, Nejjari, and de las Heras (2008), Meskin and Khorasani (2009), Wang, Wang, and Shi (2009), Narasimhan, Vachhani, and Rengaswamy (2008), Ducard (2009) and references therein. The isolation of the fault can, in some cases, be done using a similar approach, i.e., by designing filters for families of faults, and identifying the most likely fault as that associated to the filter with smaller residuals.

The main idea in such architectures stems from the design of filters that are more sensitive to faults than to disturbances and model uncertainty. This can be achieved, for instance, by using geometric considerations regarding the plant, as in Massoumnia (1986a, 1986b), Longhi and Moteriù (2009), Bokor and Balas (2004), or by optimizing a particular norm minimization objective, such as the $\mathcal{H}_\infty$- or l_1-norm—see Edelmayer, Bokor, and Keviczky (1994), Frank and Ding (1997), Niemann and Stoustrup (2001), Marcos, Ganguli, and Balas (2005), Collins and Tinglun (2001). The latter approach provides, in general, important robustness properties, as stressed in Edelmayer et al. (1994), Mangoubi, Appleby, Verghese, and Vander Velde (1995), Patton and Chen (1997) and Esteban (2004), by explicitly accounting for model uncertainty.

The FDI strategy proposed in this paper uses a different philosophy. Instead of identifying the most likely model of the faulty plant, one discards models that are not compatible with the observations. As shown in the sequel, this method guarantees that there will not be false alarms, as long as the model of the non-faulty plant remains valid. I.e., if the assumptions regarding the bounds on the exogenous disturbances are not violated, and the model of the dynamics of the plant is valid, then no fault is declared. Moreover, one need not

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compute the decision threshold used to declare whether or not a fault has occurred. To this end, the technique introduced in Rosa, Silvestre, Shamma, and Athans (2009, 2010), which is based upon the use of set-valued observers (SVOs) – see Witsenhausen (1968), Schweppe (1968, 1973), Milanese and Vicino (1991) and Shamma and Tu (1999) and references therein for an overview on SVOs – is extended.

In this paper, an application example of a new FDI method based on SVOs is provided, and the performance of the aforementioned technique when applied to the detection of faults in an aircraft is addressed. The performance of the approach is assessed in simulation, by deliberately generating faults in the nonlinear aircraft model. The key criteria of this evaluation are the time required to diagnose a failure, and the robustness of the method against model uncertainty and exogenous disturbances.

The remainder of this paper is organized as follows: Section 2 introduces the robust SVOs that are going to be used for FDI; Section 3 presents the methodology to design FD filters using SVOs, and discusses some of the approaches to modeling several types of faults; Section 4 extends the ideas in Section 3 for isolating the faults; Section 5 presents the full nonlinear longitudinal aircraft dynamic model and the corresponding LPV approximation; simulation results for the nonlinear dynamic model of the longitudinal dynamics of an aircraft are presented in Section 6; and finally, Section 7 summarizes some of the conclusions regarding this paper.

2. Set-valued observers

As shown in the sequel, the problem of "disqualifying" dynamic models of a system can be tackled using set-valued observers (SVOs)—see Witsenhausen (1968), Schweppe (1968, 1973) and Milanese and Vicino (1991). One assumes that the non-faulty plant can be represented by an uncertain (possibly time-varying) discrete-time linear system, with uncertain initial conditions, and excited by bounded but unknown exogenous disturbances, i.e.,

$$\begin{cases} \boldsymbol{x}(k+1) = (\mathbf{A}(k)+\mathbf{A}_\Delta(k))\boldsymbol{x}(k)+\mathbf{L}_d(k)\boldsymbol{d}(k)+(\mathbf{B}(k)+\mathbf{B}_\Delta(k))\boldsymbol{u}(k), \\ \boldsymbol{y}(k) = (\mathbf{C}(k)+\mathbf{C}_\Delta(k))\boldsymbol{x}(k)+\boldsymbol{n}(k), \end{cases} \tag{1}$$

where $\boldsymbol{x}(k)\in\mathbb{R}^n$, $\boldsymbol{d}(k)\in\mathbb{R}^{n_d}$, $\boldsymbol{u}(k)\in\mathbb{R}^{n_u}$, $\boldsymbol{y}(k)\in\mathbb{R}^{n_y}$, $\boldsymbol{x}(0)=\boldsymbol{x}_o$, $\boldsymbol{x}_o\in X(0)$, $\boldsymbol{d}(k)$ with $\|\boldsymbol{d}(k)\| := \max_i|\boldsymbol{d}_i(k)| \le 1$ are the disturbances, $\boldsymbol{n}(k)$ with $\|\boldsymbol{n}(k)\| := \max_i|n_i(k)| \le \overline{n}$ is the sensors noise, $\boldsymbol{u}(k)$ is the control input, $\boldsymbol{y}(k)$ is the measured output, $\boldsymbol{x}(k)$ is the state of the system and $X(0) := \mathrm{Set}(\mathbf{M}_o,\boldsymbol{m}_o)$, where, for any positive integers ℓ_1 and ℓ_2, and for any matrix $\mathbf{M}\in\mathbb{R}^{\ell_1\times\ell_2}$ and vector $\boldsymbol{m}\in\mathbb{R}^{\ell_1}$,

$$\mathrm{Set}(\mathbf{M},\boldsymbol{m}) := \{\boldsymbol{q} : \mathbf{M}\boldsymbol{q} \le \boldsymbol{m}\} \tag{2}$$

represents a convex polytope. In $X(0)$, $\mathbf{M}_o$ is a known matrix and $\boldsymbol{m}_o$ is a known vector, and are used to describe the uncertainty regarding the initial state of the system. The matrix $\mathbf{A}(k)+\mathbf{A}_\Delta(k)$ models the uncertain dynamics of the system, at time k, with $\mathbf{A}(k)$ known and $\mathbf{A}_\Delta(k)$ uncertain, as further described in the sequel. A similar structure is assumed to the input and output matrices, i.e., $\mathbf{B}(k)+\mathbf{B}_\Delta(k)$ and $\mathbf{C}(k)+\mathbf{C}_\Delta(k)$, respectively. The matrix $\mathbf{L}_d(k)$ describes the direction upon which the disturbances can act, at time k, and is also assumed known. Moreover, it is assumed that the matrices of the dynamics of (1) are uniformly bounded, i.e., there exists $\alpha<\infty$ such that

$$\|\mathbf{A}(k)\|,\|\mathbf{A}_\Delta(k)\|,\|\mathbf{L}_d(k)\|,\|\mathbf{B}(k)\|,\|\mathbf{B}_\Delta(k)\|,\|\mathbf{C}(k)\|,\|\mathbf{C}_\Delta(k)\| \le \alpha,$$

for all k, where, for a matrix $\mathbf{M}$, $\|\mathbf{M}\|$ denotes the maximum singular value of $\mathbf{M}$. The elements of vector $\boldsymbol{v}(k)$ are represented as $v_i(k)$, so that $\boldsymbol{v}(k) = [v_1(k), v_2(k), \ldots, v_m(k)]^{\mathrm{T}}$. The concatenation of a sequence of vectors $\boldsymbol{v}(k)$, $\boldsymbol{v}(k-1)$, $\cdots$, $\boldsymbol{v}(k-N+1)$ is denoted by

$$\boldsymbol{v}_N = \begin{bmatrix} \boldsymbol{v}(k) \\ \vdots \\ \boldsymbol{v}(k-N+1) \end{bmatrix}.$$

For the sake of simplicity, $\boldsymbol{v}$ is used instead of $\boldsymbol{v}_N$ whenever N can be inferred from the context. Also, for a matrix $\mathbf{M}\in\mathbb{R}^n$, let

$$\begin{bmatrix} \mathbf{M} \\ \star \end{bmatrix} := \begin{bmatrix} \mathbf{M} \\ -\mathbf{M} \end{bmatrix}.$$

Furthermore, it is assumed that

$$\mathbf{A}_\Delta(k) = \mathbf{A}_1(k)\Delta_1(k)+\mathbf{A}_2(k)\Delta_2(k)+\cdots+\mathbf{A}_{n_A}\Delta_{n_A}(k),$$

for $|\Delta_i(k)|\le 1, i=1,\ldots,n_A$. The scalars $\Delta_i(k)$, $i=\{1,\ldots,n_A\}$, represent parametric uncertainties, while the matrices $\mathbf{A}_i(k)$, $i=\{1,\ldots,n_A\}$, are the directions which those uncertainties act upon. For the time being, it is assumed that

$$\mathbf{B}_\Delta(k)\equiv 0, \quad \mathbf{C}_\Delta(k)\equiv 0, \quad k\ge 0.$$

This assumption will be dropped in the sequel, and is considered here just for the sake of clarity.

The goal here is to find $\boldsymbol{x}(k+1)$, based upon (1) and with the additional knowledge that $\boldsymbol{x}(k)\in X(k)$,$\boldsymbol{x}(k-1)\in X(k-1)$, $\ldots$,$\boldsymbol{x}(k-N)\in X(k-N)$ for some finite N. It is further required that for all $\boldsymbol{x}\in X(k+1)$, there exists $\boldsymbol{x}(k)\in X(k)$ such that the observations are compatible with (1). In other words, $X(k+1)$ should be the smallest set containing all the solutions to (1). A procedure for discrete time-varying linear systems was introduced in Shamma and Tu (1999), and preliminary results of the extension of this technique to uncertain plants were presented in Rosa et al. (2009, 2010).

However, for plants with uncertainties, the set $X(k+1)$ is, in general, non-convex, even if $X(k)$ is convex. Thus, it cannot be represented by a linear inequality as in (2). The approach suggested in Rosa et al. (2009), which assumes that $\mathrm{rank}(\mathbf{A}_i(k))=1$ for all $i\in\{1,\ldots,n_A\}$, is to overbound this set by a convex polytope, therefore, adding some conservatism to the solution.

It is presented, in the sequel, a different approach, which does not require the rank assumption on the $\mathbf{A}_i(k)$ matrices, and that reduces to the procedure in Rosa et al. (2009) whenever such a rank condition is verified. This approach amounts to solving (1) for the vertices of the hyper-cube defined by $|\Delta_i(j)|\le 1, i=1,\ldots, n_A$, and $j=k-N+1,\ldots,k$, as explained next.

Indeed, consider a realization of (1) where $\Delta_i(j)=\Delta_i^*(j)$, $i=1,\ldots,n_A$, and $j=k-N+1,\ldots,k$, and denote by $\mathbf{A}_{\Delta^*}(j)$ the corresponding uncertainty maps, i.e.,

$$\mathbf{A}_{\Delta^*}(j) = \mathbf{A}_1(j)\Delta_1^*(j)+\mathbf{A}_2(j)\Delta_2^*(j)+\cdots+\mathbf{A}_{n_A}(j)\Delta_{n_A}^*(j).$$

Then, the technique in Shamma and Tu (1999) can be used to design an SVO which computes a set-valued estimate of the state of the plant, by noting that (1) with $\Delta_i(j)=\Delta_i^*(j), i=1,\ldots,n_A$, and $j=k-N+1,\ldots,k$, is equivalent to stating that there exist $\boldsymbol{x}(k+1),\ldots,\boldsymbol{x}(k-N+1)$, $\boldsymbol{y}(k)$, and $\boldsymbol{d}(k),\ldots,\boldsymbol{d}(k-N+1)$, such that,

$$\mathbf{P}(k)\begin{bmatrix} \boldsymbol{x}(k+1) \\ \boldsymbol{x}(k) \\ \boldsymbol{x}(k-1) \\ \vdots \\ \boldsymbol{x}(k-N+1) \\ \boldsymbol{d}(k) \\ \boldsymbol{d}(k-1) \\ \vdots \\ \boldsymbol{d}(k-N+1) \end{bmatrix} \le \begin{bmatrix} \mathbf{B}(k)\boldsymbol{u}(k) \\ \star \\ \mathbf{F}_k^1\mathbf{B}(k-1)\boldsymbol{u}(k-1)+\mathbf{B}(k)\boldsymbol{u}(k) \\ \star \\ \vdots \\ \mathbf{F}_k^{N-2}\mathbf{B}(k-N+1)\boldsymbol{u}(k-N+1)+\cdots+\mathbf{B}(k)\boldsymbol{u}(k) \\ \star \\ \mathbf{1} \\ \vdots \\ \mathbf{1} \\ \tilde{\boldsymbol{m}}(k) \\ \vdots \\ \boldsymbol{m}(k-N) \end{bmatrix}$$

$$=: \boldsymbol{p}(k), \tag{3}$$