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Stress analysis of elastic coated solids in point contact

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ABSTRACT

In this paper, a point contact model of a rigid ball and an elastic coated solid is presented by combining the traditional contact model and the influence coefficients (ICs). The ICs are obtained from frequency response functions with a conversion method based on FFT. The validity and accuracy of present model are verified by comparisons of the solutions obtained by the present model with the analytical solution for Hertz contact and with the pioneers' result for non-Hertzian contact. The effect of coating thickness on the stress field is numerically investigated and the preferable coating thickness is drawn.

1. Introduction

Solid coatings, such as Ag, DLC and MoS_2, are widely employed to improve tribological performance and service life of tribo-parts. More and more tribo-parts, such as gear, bearings are coated, especially for those working under severe performing conditions [1–3]. A stress analysis of elastic coated solids in point contact is essential for the design and application of coatings, since the contact substantially can be non-Hertzian depending on the elastic properties of the coating and the substrate.

Theoretical analysis of the mechanics of layered solids, linearly elastic theory used various integral transform techniques to produce solutions [4]. The general theories of the stresses and displacements of layered systems were structured by Burmister for analyzing the contact under prescribed axially symmetric surface normal loading [5,6]. Chen extended the applications to both axisymmetric and non-axisymmetric normal surface loading [7,8]. O'Sullivian and King studied the contact of coated materials using Papkovich–Neuber potentials [9]. Nogi and Kato applied the FFT to speed up the calculation based on the work of O'Sullivian and King [10]. The Papkovich–Neuber potentials were further employed to study the sliding contact or partial slip contact problem for solids coated with monolayer or multilayer [11–13]. The finite element method (FEM) was also used to analyze the contact problem of coated solids. Komvopoulos [14] established an elastic contact model between an elastic solid coated with a hard coating and a rigid cylinder with FEM, and the effect of the coating thickness was investigated. Kral et al. employed FEM to study the contact of elastic–plastic layered medium subjected to repeated indentation by a rigid sphere [15] and combined normal and sliding indentation [16]. Ye and Komvopoulos [17] established a finite element contact model between elastic–plastic layered media and an elastic sphere considering thermo-mechanical surface loading. Yang and Komvopoulos [18] investigated a plane strain problem of elastic–plastic solid subject to dynamic load with FEM. Gong and Komvopoulos [19] used FEM to model the surface crack of coated solid in plane strain contact. Song and Komvopoulos established a finite element adhesive contact model of elastic–plastic coated solid by using a nonlinear spring element to model the surface adhesion [20] and investigated the delamination of an elastic film during adhesive contact loading and unloading [21]. Kang et al. [22] also adopted FEM contact model to analyze the failure mechanism of plasma sprayed AT40 coatings under different rolling–sliding contact conditions. It was also reported that the equivalent inclusion method (EIM), which was often employed to analyze contact considering inhomogeneities, also can be used to solve the contact problem of coated solids [23]. The contact of coated solids has been extensively studied in the past several decades; however, there is still a shortage of systematic research on the effect of coating's stiffness and thickness on the stress field in subsurface for both hard and soft coatings in point contact. Furthermore, the engineering application of coatings is still in a trial-type stage with a certain blindness and randomness due to lack of accepted design criteria, although lots of efforts have been made to develop design and optimization technology of coating depositing for improving the performance of anti-wear, anti-friction and fatigue strength, etc. [24–27].

In this paper, a numerical contact model of coated solids in point contact is built based on the traditional contact model and the ICs of surface displacement and stresses of coating–substrate system. The ICs are obtained from their corresponding frequency response

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functions by using a conversion method based on FFT. Conjugate gradient method (CGM) and discrete convolution–fast Fourier transformation (DC–FFT) are employed to obtain the contact pressure distribution and stresses in the subsurface. The comparisons of the solutions obtained by present model with the analytical solution for Hertz contact and with the pioneers' research result for non-Hertzian contact are performed to verify the validity and accuracy of the present contact model. With the purpose of providing useful insight for design of coating–substrate system and exploring ways to reduce the risk of the initiation and propagation of cracks and delamination of coating from its substrate, more attentions are paid to the effect of coating thickness on the distribution of transverse stress σ_{xx} on the surface or at the base of the coating and shear stress σ_{xz} on the interface. Since those stresses are believed to be responsible for the initiation and propagation of cracks at the tail of the contact zone on the surface or at the base of the coating, and the delamination of coating from its substrate, respectively. These failure modes of coatings under contact load as shown in Fig. 1 have been traditionally recognized [28–30].

2. Theory

2.1. Description of the contact model

According to [31], a rigid ball brought into contact with a coated solid by an external normal load W as shown in Fig. 2(a), can be described as follows:

$$\begin{aligned} &g(x,y)=0, p(x,y)>0 && \in \Omega \\ &g(x,y)>0, p(x,y)=0 && \notin \Omega \\ &\int_{x_b}^{x_e}\int_{y_b}^{y_e} p(x,y)dxdy = W \\ &g(x,y)=g^0(x,y)+u^z(x,y)-\delta \end{aligned} \quad (1)$$

where $g(x,y)$ is the gap between the surfaces of the two bodies in contact after the external normal load W is applied, $g^0(x,y)$ is the initial gap before the external normal load W is applied, $u^z(x,y)$ is the surface normal displacement, δ is the normal approach and Ω is the contact area.

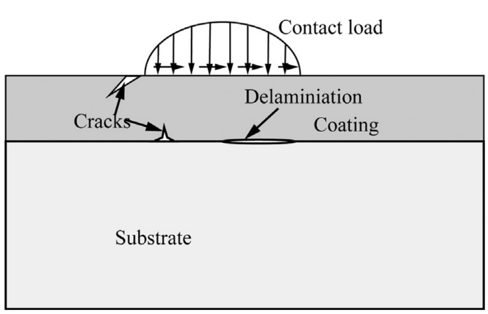

Fig. 1. Common coating failure modes.

In order to employ numerical method to obtain the contact pressure, a rectangular domain $[-2a, 2a] \times [-2a, 2a]$ is selected and uniformly divided into $N_x \times N_y$ surface patch elements centered on the grid nodes. Here a is the Hertz contact radius, N_x and N_y are the number of elements in x and y direction, respectively. The contact pressure distribution is approximated by a piecewise constant function that is uniform within each surface element. Then Eq. (1) can be converted into a discretized form as follows:

$$\begin{aligned} &g_{i,j}=0, p_{i,j}>0 && \in \Omega \\ &g_{i,j}>0, p_{i,j}=0 && \notin \Omega \\ &\Delta_x \Delta_y \sum_{i=0}^{N_x-1}\sum_{j=0}^{N_y-1} p_{i,j} = W \\ &g_{i,j}=g^0{}_{i,j}+u^z{}_{i,j}-\delta \end{aligned} \quad (2)$$

where Δ_x and Δ_y are the discretization size in x and y directions.

In Eq. (2), the surface normal displacement $u^z_{i,j}$ at the grid node $S_{i,j}$ can be obtained by

$$u^z_{i,j} = \sum_{k=0}^{N_x-1}\sum_{l=0}^{N_y-1} K_{i-k,j-l}\, p_{k,l} \quad (3)$$

where $K_{i-k,j-l}$ are the ICs of surface normal displacement. If the substrate is uncoated, an explicit formula for $K_{i-k,j-l}$ can be given by applying Boussinesq and Cerruti solutions [31,32].

Several different iteration schemes have been proposed to solve Eq. (2), and the iteration scheme based on CGM [33,34] is employed here for its comparatively high rate of convergence and explicit iteration format which makes it compatible with the fast multi-summation method, namely, the DC–FFT [35].

After the contact pressure is obtained, the stress field in the subsurface can be obtained as follows:

$$\sigma_{rs}\left(x_i, y_j, z\right) = \sum_{k=0}^{N_x-1}\sum_{l=0}^{N_y-1} SN^{rs}_{i-k,j-l}(z)p_{k,l} + \sum_{k=0}^{N_x-1}\sum_{l=0}^{N_y-1} ST^{rs}_{i-k,j-l}(z)q_{k,l} \quad (4)$$

where $SN^{rs}_{i-k,j-l}(z)$ and $ST^{rs}_{i-k,j-l}(z)$ are the stresses ICs of point at z depth lying directly below grid node $S_{i,j}$ due to surface normal traction $p_{k,l}$ and tangential traction $q_{k,l}$, respectively. And $q_{k,l} = f p_{k,l}$ by using Coulomb's friction law and denoting the friction coefficient with f. If the solid is uncoated, explicit formulas for $SN^{rs}_{i-k,j-l}(z)$ and $ST^{rs}_{i-k,j-l}(z)$ also can be obtained by applying Boussinesq and Cerruti solutions. The detail can be seen in [32].

For elastic solid with a coating bonded to its surface, the explicit formulas for ICs of displacement and stress components are not available. However, they can be obtained from their corresponding frequency response functions by a conversion method based on FFT [36,37].

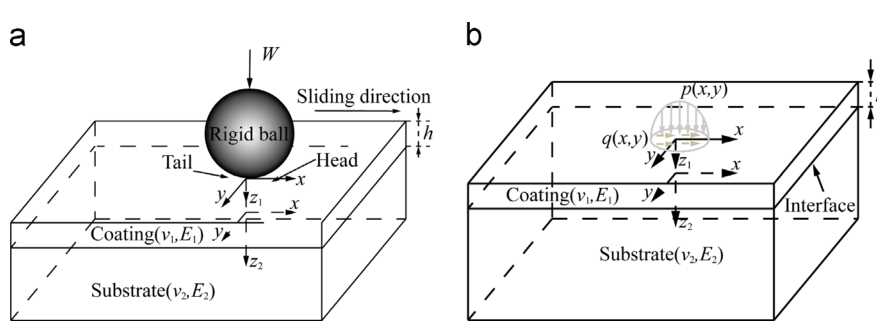

Fig. 2. (a) A rigid ball and a coated solid in point contact, (b) coated solid subjected to arbitrary surface normal traction and tangential traction along the x direction.