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Design of modulated and demodulated controllers for flexible structures ☆

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Abstract

We propose a novel method for controlling vibrations within a resonant structure equipped with piezoelectric transducers. The scheme uses a parallel connection of modulated and demodulated controllers, each designed to damp the transient oscillation corresponding to a single mode. This technique allows multiple modes to be controlled with a single actuator. A simulation example is presented and design considerations for the scheme are discussed. Experimental results obtained from a piezoelectric laminate cantilever beam confirm the theoretical analysis.

Keywords: Amplitude modulation; Vibration control; Active control; Design issues

1. Introduction

There has been significant research interest in utilising piezoelectric transducers for vibration mitigation in resonant mechanical structures, see e.g. Moheimani and Goodwin (2001), Moheimani (2003), Fuller, Elliot, and Nelson (1996), Clark, Saunders, and Gibbs (1998), Moheimani, Halim, and Fleming (2003). By bonding piezoelectric materials to the surface of a resonant structure, these transducers can be used for vibration control, where they can be deployed as actuators, as sensors or both. For that purpose, several control algorithms have been proposed, see e.g. Hagood, Chung, and von Flotov (1990), Hagood and von Flotov (1991), Lazarus, Crawley, and Bohlman (1991), Moheimani, Fleming, and Behrens (2001).

Given the fact that piezoelectric transducers can be accurately described with linear models over a significant operating range (Moheimani, 2000), both active and passive LTI (linear time invariant) methods have been proposed, see e.g. the survey in Moheimani (2003). Restricting controllers to be LTI is certainly attractive since, in this case, the design problem can be cast in the well-studied LTI control systems framework, see e.g. Goodwin, Graebe, and Salgado (2001). On the other hand, it is well known that resonant systems are not always easy to control with LTI methods (Goodwin et al., 2001; Serón, Braslavsky, & Goodwin, 1997). As a consequence, depending upon the application, non-LTI methods may be worth studying, see e.g. Corr and Clark (2003) for work on non-linear switching methods within this context.

In the present work, we propose a time varying (more precisely, periodic) vibration control method for resonant systems. It makes use of the fact that signals within a resonant structure are of an oscillatory nature, and hence concentrate their energies around a set of discrete frequencies, which correspond to the modes of the mechanical system. The method described here utilises concepts from amplitude-modulated communication systems (Haykin, 2001) in order to isolate and shift the spectrum of the high-frequency oscillations down to the *baseband*. It then operates on these low-frequency signals. This corresponds to controlling the envelope of the

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oscillations and has strong conceptual and computational advantages, when compared to operating on high-frequency signals.

Our methodology is based upon so-called *modulated* and *demodulated* control methods, which have been proven to be effective in a number of applications, see e.g. Bode (1945), Chen, M'Closkey, Tran, and Blaes (2005), Chang (1993), Leland (2001), Gerber (1978). Specifically, in relation to vibration suppression of flexible structures, an indirect method has been proposed by Chang (1993). It utilises the plant output as the modulation signal which translates energy into the baseband. The method emulates modulation with exogenous signals by relying upon a, rather ad hoc, non-linear gain adjust module. Implicit in Chang (1993) is the assumption that the controlled plant output always has enough energy at the natural frequencies of the structure so as to provide an adequate modulation signal. This certainly constitutes a limiting factor, since the aim of the controller resides precisely in vibration suppression.

As shown by Chang (1993), one of the advantages of using modulation and demodulation is that it allows the modes of the structure to be controlled using a low-bandwidth controller. This is particularly useful in the control of high-frequency vibration modes.

In the present paper, we propose to utilise modulated and demodulated control with exogenous modulation signals. This allows one to control several modes with a single piezoelectric actuator in a simple manner. It also overcomes the above-mentioned limitations of the output-modulation based method of Chang (1993). We study the behaviour of the resulting closed loop system using several linear models each of which yields different insights. Indeed, the closed loop behaviour turns out to be governed by a trade-off between attenuation and stability margin.

Our work extends Lau, Goodwin, and M'Closkey (2004b) to the control of multiple modes within a resonant structure, which we assume fixed and known. Preliminary ideas can also be found in our conference contribution (Lau, Quevedo, Goodwin, & Moheimani, 2004a).

The remainder of the paper is organised as follows. In Section 2, we describe the system to be controlled. Then, in Section 3, we investigate the application of modulated control to vibration damping. Section 4 elucidates design considerations, and Section 5 documents experimental studies. Section 6 concludes the paper.

2. Piezoelectric beam model

Piezoelectric materials can transform electrical energy into mechanical energy and vice versa. Thus they can be deployed as actuators, as sensors or both. More precisely, due to their permanent dipole nature, piezoelectric materials strain when exposed to an electric field and conversely produce an electric charge when strained. More details can be found e.g. in Fuller et al. (1996), Moheimani (2003).

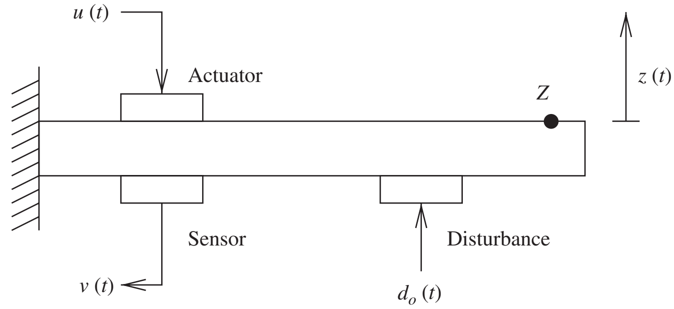

Fig. 1. Piezoelectric laminate beam.

In the present work, we concentrate upon the piezoelectric laminate beam schematised in Fig. 1. In this configuration, the beam is fixed at one end and free at the other. Three piezoelectric patches are mounted on the beam, two being collocated. As shown in Fig. 1, one patch is used as an actuator, i.e., it transforms the voltage $u(t)$ into strain. Another patch functions as a sensor, providing a voltage $v(t)$, which depends upon the (local) beam deflection. The beam is also equipped with a piezoelectric actuator, which represents disturbances. The disturbance is modelled by $d_0(t)$, which exerts a moment on the beam.

The overall goal of our work is to provide a controller for the system depicted in Fig. 1 which, based upon the measured signal $v(t)$ and by manipulating the signal $u(t)$, mitigates the vibration caused by $d_0(t)$. We also examine the displacement $z(t)$, which corresponds to some other point Z on the beam.

The laminate beam of Fig. 1 is governed by a partial differential equation which can be solved in a variety of ways, see e.g. Fuller et al. (1996), Moheimani et al. (2003), Pota, Moheimani, and Smith (1999). In the present work, we will utilise the following model, taken from Moheimani (2003):

$$\begin{aligned} v(t) &= G_{vu}(s)u(t) + G_{vd}(s)d_0(t), \\ z(t) &= G_{zu}(s)u(t) + G_{zd}(s)d_0(t). \end{aligned} \tag{1}$$

Here, $G_{vu}(s)$, $G_{vd}(s)$, $G_{zu}(s)$ and $G_{zd}(s)$ are linear time invariant transfer functions, which reflect the resonant nature of the beam. In principle, they contain an infinite number of natural modes, being of the form

$$\sum_{i=1}^{\infty} \frac{\gamma_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}, \tag{2}$$

where ω_i are the resonance frequencies and ζ_i are their (uncontrolled) damping factors. The scalars γ_i depend upon the position of the piezoelectric patches and, in the collocated case of Fig. 1, are always non-negative.

For control design purposes it is convenient to truncate the model structure (2) such that only M modes are considered. Thus, the relationship between $u(t)$ and $v(t)$ can