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Slip and wear at a corner with Coulomb friction and an interfacial strength

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ABSTRACT

Traditional analyses of slip at corners of contacts, based on linear elasticity and a Coulomb friction law, are very sensitive to the details of local geometries, owing to the effects of elastic singularities. Following the use of cohesive-zone models to address such issues in mode-II fracture, we present analyses of slip and wear at corners of contacts when a finite interfacial shear strength is incorporated with a Coulomb friction law. We show that the concept of an instantaneous cohesive-length scale, borrowed from the field of fracture mechanics, can be used to describe the nature of stress fields around corners, and defines when linear-elasticity and Coulomb friction can provide an accurate description of the interfacial behavior. We also show that the sensitivity of slip analyses to geometrical details decreases when the cohesive-length scale increases. We also show that the cohesive strength of an interface plays a crucial role in the propagation of a wear scar across an interface. If only Coulomb slip is assumed to occur, a wear scar may not progress beyond the original stick-slip boundary. If a finite interfacial shear strength is introduced into the analysis, the wear scar can propagate along the interface.

1. Introduction

The life of many engineering components can be limited by fretting wear [1] induced by cyclic slip between two contacting surfaces [2]. This wear is often initiated at the corners of contacts, where there are high stresses that can be singular in elastic analyses. However, an assumption of Coulomb friction in these analyses leads to a prediction of slip and wear only if the coefficient of friction, μ is low enough, with the critical value of μ depending on the details of the corner geometry. Similar issues of sensitivity to local geometry are inherent for problems of crack propagation in interfacial fracture mechanics. In that field, cohesive-zone models, which incorporate the concept of a finite interfacial strength, have been found to be useful tools to resolve some of the unrealistic complexities associated with singular stresses, while retaining the general features of fracture mechanics that make it useful at larger scales [3–5]. In this paper we apply the insight provided by the field of interfacial fracture mechanics to interfacial slip, showing that the assumption of a finite shear-strength changes the slip and wear behavior at corners in important ways.

The contact across an interface between two bodies can be described by three regimes of behavior that depend on the geometry and the loads [6]. The first regime is full-stick, where the interface is effectively bonded across its entire length, and the two bodies act as a single entity. The second regime is full-slip, where there is relative motion between the two bodies along the entire interface. The third regime is partial-slip, where the two materials slide relative to each other along some parts of the interface, and are effectively bonded along others. Wear is associated with the energy dissipated by sliding [6,7], and the analysis of wear requires modeling the relative slip along the interface between the two bodies. The full-stick and full-slip regimes can be relatively easy to describe; partial-stick can provide more of a challenge. However, this last phenomenon is an important aspect of wear at the corners of contacts and, therefore, forms the focus of this study.

Coulomb's law is a common criterion used to determine when slip occurs. This law states that the magnitude of the interfacial shear stress, q, is limited by the product of the local applied pressure across the interface, p, and the coefficient of friction, μ:

$$|q| \le \mu p. \quad (1)$$

Slip occurs if this condition cannot be satisfied without allowing a relative shear displacement across the interface. The coefficient of friction is generally assumed to be a constant that is characteristic

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of the interface; if it is assumed to vary, non-linear effects are introduced [8].

Coulomb's law allows for the possibility of an arbitrarily high interfacial shear stress, if the local pressure is high enough. Indeed, Coulomb slip at a corner results in singular shear stresses. This unphysical result can be avoided by assuming that the magnitude of the shear stresses is limited by an interfacial shear strength, $\hat{\tau}$, that is independent of local pressure [9,10]. For example, the shear strength of the contacting materials could provide an upper bound to this parameter. Local equilibrium then requires a second condition that

$$|q| \le \hat{\tau}. \quad (2)$$

This concept of a single-valued interfacial shear strength is commonly used in fiber-composite models [11,12], as well as in thin-film and composite-laminate cracking problems [3,13].

The stresses along an interface near a corner are generally singular for elastic bodies in contact. The strength of the singularity depends on the details of the geometry, and is the same for both the shear stress and the pressure [14–16]. This means that the ratio of the shear to normal stress is constant near the corner, no matter how high the stresses are. So, depending on the magnitude of the friction coefficient, either slip by the Coulomb criterion occurs everywhere within the singular region, or there is complete sticking within the singular region. Partial slip occurs when the Coulomb condition is met within the singular field, but not outside it. However, the stress field associated with Coulomb slip is still singular, both the contact pressure and the shear stress increase without limit as the corner is approached.

The purpose of this paper is to examine how the assumption of a limiting value for the interfacial shear stress, as given by Eq. (2), affects slip at the corner of contacts, and how this might influence the evolution of wear. In the next section we give a brief summary of how the interfacial stresses near corners depend on geometry and slip conditions. We then show how these stress distributions and the slip are affected by the assumption of a finite interface strength. This is followed by a demonstration of how the sensitivity of slip to details of the corner geometry is reduced by invoking a finite interfacial strength. The final part of the paper demonstrates how the assumption of a finite interface strength can have a significant effect on the evolution of a wear scar, allowing it to propagate across an interface, rather than arresting at the initial slip-stick boundary.

2. Background

2.1. No slip

In this paper, we consider two elastic bodies with the same properties, contacting each other along flat surfaces. Fig. 1 shows a magnified view near the corner of the contact for such a system. If the interface does not satisfy either of the two slip conditions of Eqs. (1) and (2), the stresses are given by the elastic solution for a wedge. Close to the corner, the stresses are singular. For example, along a line that bisects the corner (Fig. 1), the singular components of the normal and shear stresses are given by [17,18]

$$p_{\theta\theta}(r) = K_I r^{\lambda_I - 1},$$
$$q_{r\theta}(r) = K_{II} r^{\lambda_{II} - 1}, \quad (3)$$

where r is the distance from the corner, K_I and K_{II} are the mode-I (symmetrical) and mode-II (anti-symmetrical) stress-intensity factors, and the strengths of the singularities, λ_I and λ_{II}, depend on the exterior angle, ϕ. The two stress-intensity factors depend on the detailed geometry of the corner, but they also depend on the

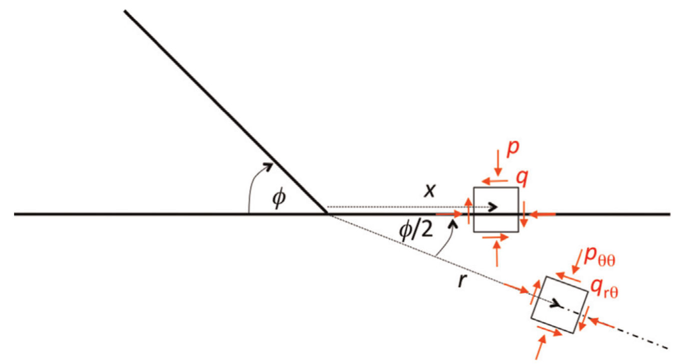

Fig. 1. Geometry of a corner with an exterior angle ϕ, and an interface along which sliding can occur.

macroscopic geometry, and on the applied loads. They are analogous to those used in fracture mechanics, and describe the effects of the geometry and loads.[1]

Equation (3) describes the singular stresses along a line that bisects the exterior angle. Of particular interest are the normal pressure, p, and shear stress, q, along the interface [18]:

$$p(x) = f_1 K_I x^{\lambda_I - 1} + f_2 K_{II} x^{\lambda_{II} - 1},$$
$$q(x) = f_1 g_1 K_I x^{\lambda_I - 1} + f_2 g_2 K_{II} x^{\lambda_{II} - 1}, \quad (4)$$

where x is the distance along the interface (Fig. 1), and f_1, f_2, g_1 and g_2 are functions of ϕ. The geometry we will generally consider in this paper is one for which $\phi = 90°$. For this case, the stresses along the interface are given by [18]

$$p(x) = 0.7303 K_I x^{-0.4555} - 1.0873 K_{II} x^{-0.0915},$$
$$q(x) = 0.3966 K_I x^{-0.4555} + 0.2381 K_{II} x^{-0.0915}. \quad (5)$$

2.2. Coulomb slip

A comparison between Eqs. (4) and (1) reveals that slip will always occur in the singular region at the corner of a contact if $\mu < g_1(\phi)$, since λ_I dominates the stress field close to the corner. In particular, Eq. (5) shows that slip will occur if $\mu < 0.543$ for the right-angled geometry considered in this paper. If the Coulomb slip condition is met within the singular region, the asymptotic stress field develops a different singularity, λ_s, that is a function of both ϕ and μ. The interfacial stresses close to the corner are then given by [18]

$$p(x) = K_s x^{\lambda_s - 1},$$
$$q(x) = \mu K_s x^{\lambda_s - 1}, \quad (6)$$

where K_s is a stress-intensity factor that depends on the geometry and loads. For a right-angled corner, λ_s is given by the solution to [19,20]

$$[\sin^2(\pi\lambda_s/2) - \lambda_s^2]\cos(\pi\lambda_s) + (1/2)\sin^2(\pi\lambda_s)$$
$$+ \mu\lambda_s(1 + \lambda_s)\sin(\pi\lambda_s) = 0. \quad (7)$$

[1] As can be seen from the definition of the singular stress field, the stress-intensity factors usually used in the fracture mechanics literature differ from those used in the friction literature by a factor of $\sqrt{2\pi}$.