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Robust control of the missile attitude based on quaternion feedback

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Abstract

In this paper, a robust control scheme based on the quaternion feedback for attitude control of missiles employing thrust vector control is proposed. The control law consists of two parts: the nominal feedback part and an additional term for ensuring robustness to the plant uncertainties. For the proposed control scheme, a stability analysis is given and the performance is shown via computer simulation.

Keywords: Quaternion feedback; Attitude control; Uncertainty; Stability analysis

1. Introduction

In general, most attitude control schemes of tactical missiles are based on the Euler angle feedback concept. However, modern satellites or spacecrafts have a trend toward using quaternion feedback instead of Euler angle feedback (Weiss, 1993; Wie & Barba, 1985; Wie, Weiss, & Arapostathis, 1989). As described in the references, quaternion control enables the attitude change along the shortest path by matching the control torque vector to the eigenaxis which is not possible with Euler angle control because Euler angles are based on the concept of sequential rotation. Moreover, Wie et al. (1989) showed that the quaternion feedback control system is globally stable and near-eigenaxis rotation can be achieved even in the presence of initial body rate and inertia matrix uncertainty.

However, similar research is hardly found in the area of attitude control for the tactical missiles operated in the low atmosphere. It seems due to a view that the quaternion feedback will not retain its advantage where the aerodynamic effects are not negligible. But Song et al. proposed a control scheme which might be prospective even in this case (Song, Nam, & Kim, 2000). In this paper, the results are extended to the case where the plant uncertainties exist.

Several control design methods for the uncertain dynamical systems are introduced in Barmish, Corless, and Leitmann (1983), Corless and Leitmann (1981) and Khalil (1996). Most of them require that the uncertain system satisfies the so-called "matching condition." The plant uncertainty considered here satisfies this condition. Based on this plant uncertainty model and a robust stabilization method given in Khalil (1996), an attitude controller for a vertical launch anti-submarine rocket (VLASR) model is constructed. This controller consists of two parts: a nominal feedback part and an additional term ensuring the robustness to the plant uncertainties.

This paper is organized as follows. First, missile dynamics with uncertainty and kinematics will be described. After that, a nominal controller and robust controller will be given with a stability analysis. Finally, a design example will be given with computer simulation results which show that the robust controller has better performance than the nominal one.

2. Missile dynamics with uncertainty

Missile motion is described by six degrees of freedom equations which consist of the translational and rotational

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motion equations as follows:

$$m\dot{\boldsymbol{v}} + m(\boldsymbol{\omega} \times \boldsymbol{v}) - \boldsymbol{g} = \boldsymbol{F}_a(\boldsymbol{v}) + \boldsymbol{F}_t(\boldsymbol{u}), \tag{1}$$

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = \boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{M}_a(\boldsymbol{v}, \boldsymbol{\omega}) + \boldsymbol{M}_t(\boldsymbol{u}), \tag{2}$$

where $\boldsymbol{v} = [v_1, v_2, v_3]^{\mathrm{T}}$ is the linear velocity vector, $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^{\mathrm{T}}$ is the angular velocity vector, $\boldsymbol{u} = [\delta_r, \delta_p, \delta_y]^{\mathrm{T}}$ is the control input vector, m is the mass of a missile, $\boldsymbol{J}$ is a inertia matrix, $\boldsymbol{g}$ is gravity, $\boldsymbol{F}_a$ and $\boldsymbol{M}_a$ are aerodynamic force and moment vectors, respectively, and $\boldsymbol{F}_t$ and $\boldsymbol{M}_t$ are control force and control moment vectors, respectively.

Now, in order to simplify Eqs. (1) and (2), the following assumptions are used:

(A1) Velocity and altitude of the missile are constant.
(A2) Gravity is neglected.
(A3) v_2 and v_3 are much smaller than v_1.
(A4) Missile body has the symmetrical cruciform.
(A5) $\boldsymbol{F}_t(\boldsymbol{u})$ and $\boldsymbol{M}_t(\boldsymbol{u})$ are linear in $\boldsymbol{u}$ and invertible.

Under A1–A4, the translational motion equation (1) can be written as (Hemsch & Nielsen, 1986):

$$\dot{\alpha} = \omega_2 + Z_o(Q, n, \alpha) - Z_t\delta_p,$$
$$\dot{\beta} = -\omega_3 + Y_o(Q, n, \beta) + Z_t\delta_y, \tag{3}$$

where α is the angle of attack, β is the sideslip angle, Q is the dynamic pressure, and n is Mach number. $Z_o(Q, n, \alpha)$ and $Y_o(Q, n, \beta)$ are aerodynamic coefficients, and Z_t is the control thrust coefficient. They are given by

$$Z_o(Q, n, \alpha) = \frac{QS}{mv_1} C_z(n, \alpha),$$
$$Y_o(Q, n, \beta) = \frac{QS}{mv_1} C_y(n, \beta),$$
$$Z_t = \frac{T_c}{mv_1}, \tag{4}$$

respectively, where $C_z(n, \alpha)$ and $C_y(n, \beta)$ are nondimensional aerodynamic coefficients, S is the reference area, and T_c is the magnitude of the control thrust. Eq. (3) can be rewritten in a compact form:

$$\dot{\boldsymbol{z}} = \boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\omega}) + \boldsymbol{F}_0 + \boldsymbol{E}\boldsymbol{u}, \tag{5}$$

where

$$\boldsymbol{z} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad \boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\omega}) = \begin{bmatrix} \omega_2 \\ -\omega_3 \end{bmatrix}, \quad \boldsymbol{F}_0 = \begin{bmatrix} Z_0 \\ Y_0 \end{bmatrix},$$
$$\boldsymbol{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -Z_t & 0 \\ 0 & 0 & Z_t \end{bmatrix}.$$

For the missile with cruciform configuration, $\boldsymbol{M}_a$ in Eq. (2) can be described as

$$\boldsymbol{M}_a = \begin{bmatrix} L_o \\ M_o \\ N_o \end{bmatrix} + \begin{bmatrix} L_p\omega_1 \\ M_q\omega_2 \\ M_q\omega_3 \end{bmatrix} = \begin{bmatrix} QSDC_l(n, \alpha)\sin 4\gamma \\ QSDC_m(n, \alpha) \\ QSDC_m(n, \beta) \end{bmatrix} + \frac{QSD^2}{2v_1} \begin{bmatrix} C_{lp}(n)\omega_1 \\ C_{mq}(n)\omega_2 \\ C_{mq}(n)\omega_3 \end{bmatrix} \tag{6}$$

where D is the reference length, C_l, C_m, C_{lp}, and C_{mq} are the nondimensional moment coefficients, and γ is the bank angle defined by

$$\gamma = \frac{v_2}{v_3} = \frac{\beta}{\alpha}.$$

Moreover, without loss of generality we assume

$$\boldsymbol{J} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_2 \end{bmatrix}$$

for some constants J_1, J_2. The control moment $\boldsymbol{M}_t$ is given by

$$\boldsymbol{M}_t = \begin{bmatrix} L_t\delta_r \\ M_t\delta_p \\ M_t\delta_y \end{bmatrix} = \begin{bmatrix} T_c l_y \delta_r \\ T_c l_x \delta_p \\ T_c l_x \delta_y \end{bmatrix}, \tag{7}$$

where T_c is the magnitude of the control thrust, l_x and l_y are the moment arms defined by the distance from the center of gravity to the location of the control thrust vector, and δ_r, δ_p, and δ_y are control inputs.

Now, consider the uncertainty terms by $\boldsymbol{\Delta F}_0$, $\boldsymbol{\Delta E}$, $\boldsymbol{\Delta J}$, $\boldsymbol{\Delta M}_a$, and $\boldsymbol{\Delta B}$ corresponding to $\boldsymbol{F}_0$, $\boldsymbol{E}$, $\boldsymbol{J}$, $\boldsymbol{M}_a$, and $\boldsymbol{B}$, respectively. Then, the motion Eqs. (2) and (5) can be modified as

$$(\boldsymbol{J} + \boldsymbol{\Delta J})\dot{\boldsymbol{\omega}} = \boldsymbol{\Omega}(\boldsymbol{J} + \boldsymbol{\Delta J})\boldsymbol{\omega} + \boldsymbol{M}_a + \boldsymbol{\Delta M}_a + (\boldsymbol{B} + \boldsymbol{\Delta B})\boldsymbol{u}, \tag{8}$$

$$\dot{\boldsymbol{z}} = \boldsymbol{h}(\boldsymbol{z}, \boldsymbol{\omega}) + \boldsymbol{F}_0 + \boldsymbol{\Delta F}_0 + (\boldsymbol{E} + \boldsymbol{\Delta E})\boldsymbol{u} \tag{9}$$

with the skew symmetric matrix $\boldsymbol{\Omega}$ and the input matrix $\boldsymbol{B}$ defined by

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} L_t & 0 & 0 \\ 0 & M_t & 0 \\ 0 & 0 & M_t \end{bmatrix}.$$

From the matrix inversion lemma

$$(\boldsymbol{J} + \boldsymbol{\Delta J})^{-1} = \boldsymbol{J}^{-1} - \boldsymbol{\Delta X} \tag{10}$$

with $\boldsymbol{\Delta X}$ defined by

$$\boldsymbol{\Delta X} = \boldsymbol{J}^{-1}\boldsymbol{\Delta J}(\boldsymbol{I} + \boldsymbol{J}^{-1}\boldsymbol{\Delta J})^{-1}\boldsymbol{J}^{-1}.$$

Eqs. (8) and (10) lead to

$$\dot{\boldsymbol{\omega}} = \boldsymbol{f}(\boldsymbol{\omega}, t) + \boldsymbol{\Delta H} + (\boldsymbol{G} + \boldsymbol{\Delta G})\boldsymbol{u}, \tag{11}$$