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A complete contact model of a fractal rough surface

Xiaomei Miao, Xiaodiao Huang *

School of Mechanical and Power Engineering, Nanjing University of Technology, Nanjing 210009, China

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ABSTRACT

This study suggested a revision to the contact model of a fractal rough surface, by extending the modified asperity contact model developed by Morag and Etsion, into a complete contact model of a fractal surface. According to the modified asperity model, the critical area of a single asperity was scale dependent and that the asperity's plastic to elastic mode transition agreed with classical contact mechanics. The total load, area and stiffness of a fractal rough surface were studied, and obtained by summing over the contact force, area and stiffness of the asperities at all length scales. The results revealed that the contact area depended linearly on the contact load and that the contact stiffness increased with increasing contact load. The share of plastic contact area decreased as the contact load increased. The impact of the fractal roughness parameter G and fractal dimension D on the contact stiffness was also discussed, and the results showed that the rougher the surface became, corresponding to a smaller value of D and a larger value of G, the smaller the contact stiffness was.

1. Introduction

All engineered surfaces are rough in nature, and the contact between surfaces is characterized by the interaction of asperities. This contact characterization has a significant impact on the phenomena of friction, wear, adhesion, lubrication, contact stiffness and damping. It is important to study and model the deformation behavior of asperities and rough surfaces using adequate parameters. Rough surface contact has been regarded as a stationary random process characterized by statistical parameters, such as the standard deviation of asperity heights σ, the slope σ' and the curvature σ''. Greenwood and Williamson have done pioneering work in this field [1], developing an asperity-based contact model called the GW model. Asperities are considered to have a Gaussian or exponential distribution in this model, and every asperity possesses an identical radius of curvature. In later models, the asperity curvatures are assumed to be dependent on their heights or to use fractal theories to model the multiscale nature of a real surface [2,3]. Following the pioneer work of Greenwood and Williamson, other researchers have worked in this and related fields, achieving a series of important developments. Greenwood and Tripp [4] have also assumed a Gaussian distribution of asperities and noted that the shape and relative positions of asperities have no effect on the model. Moreover, they have also proposed that two rough surfaces in contact can be simplified to an equivalent rough surface in contact with a rigid flat surface. Whitehouse and Archard [5] have proposed a WA model based on the GW model for which they assume that the curvature radius of asperities is dependent on their heights and that the profile autocorrelation function is exponential. Essentially, the GW model has focused on physical conception research, whereas the WA model has put particular emphasis on studying the contact between rough surfaces using math and random process theory. The results obtained through the two methods have not been that different. McCool [6] and Bhushan [7] have described the rough surface more precisely and complexly by adding more statistical parameters, such as the slope of asperity heights σ' and the curvature σ''. They have also accounted for the interaction between two neighboring asperities and anisotropic surfaces. Chang et al. [8] have modified the original GW model based on volume conservation when an asperity deforms plastically, while Kogut and Etsion [9] have researched the contact between a deformable hemisphere and a rigid flat surface using the finite element method. They have provided a dimensionless contact load and contact area over the increase in the interference range from full elastic through elastoplastic to fully plastic contact and also developed a static friction model for elastoplastic rough surfaces with a more practical surface height distribution [10].

However, modern roughness measurements have revealed that many engineered surfaces have a multiscale fractal nature: when a section of a rough surface is magnified, smaller scales of roughness appear [11]. In response to this finding, the assumption of statistical models stating that a surface is composed of asperities belonging to a single length scale is an oversimplification. A "fractal" description was first proposed by Archard [12], who has

* Corresponding author. Tel.: +8613327826399; fax: +86210009.
E-mail address: miaoxiaomeinjut@126.com (X. Huang).

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suggested a contact model in which smaller hemispherical asperities are superposed on a larger scale and showed that the contact load depends linearly on the contact area. Ciavarella and Demelio [13] have reviewed Archard's model for elastic multiscale contact between rough surfaces and compared it with modern fractal models. Berry and Lewis [14] have formed the initial basis for a fractal surface roughness description using the Weierstrass and Mandelbrot fractal function (the WM function). Majumdar and Bushan [15] have developed the first fractal contact models (the MB model) for real rough surfaces using the WM function. This model has been of interest to many researchers over the last twenty years. Willner [16] has applied this model to different areas of applied physics, and Ciavarella et al. [17] have investigated the elastic contact stiffness and contact resistance of rough surfaces based on this model. Komvopoulos and Yan [18] have extended this model to three-dimensional fractal surfaces, while Bora et al. [19] have developed a method to investigate the geometry of asperities in silicon Micro-electromechanical Systems surfaces at different length scales. Sahoo and Chowdhury [20,21] have analyzed the friction and wear of fractal surfaces, and Kogut and Komvopoulos [22,23] have applied the fractal surface concept to the field of electro-mechanics. Kogut and Jackson [24] have compared the statistical and fractal approaches to contact modeling, showing substantial differences between the two. Tsai et al. [25] have applied the model to sphere- and cylinder-based fractal bodies in contact with a smooth rigid flat surface. Chung and Lin [26] and Liou and Lin [27,28] have developed modified fractal contact models with variable fractal dimensions that change according to the applied load.

However, both the MB model and the related studies mentioned above show that smaller spots tend to deform plastically, whereas larger spots deform elastically. In other words, a transition from elastic to plastic contact occurs in this sequence as the load and contact area increase. This phenomenon is essentially different from classical contact mechanics and appears to be impractical, possibly because the contact area of an asperity in the MB model is equal to the truncation area at the interference and every asperity is assumed to be fully deformed. Morag and Etson [29] have proposed a modified micro contact model of asperities (the ME model) to overcome this shortcoming in the MB model. However, they have provided a contact model of only a single asperity, not a complete rough surface. The ME model have attracted many attentions of the researchers of rough surface, e.g., Jackson [30] have applied this model to an Archard-Type model [12] of rough surface, while Goedecke and Jackson [31] have applied this model to a Jackson–Streator multi-scale model [32] to investigates the resolution-dependent contact area.

The following sections present a modified contact model of a fractal rough surface, by extending the ME model to a fractal surface proposed by Majumdar and Bhushan [15]. The total load, area and stiffness of a fractal rough surface were studied, by summing over the contact force, area and stiffness of the asperities at all length scales. The critical area in our model is scale dependent and the deformation mode transition agrees with classical contact mechanics.

2. ME model

Fig. 1 illustrates the conceptual approach of the MB model [15]. Two contacting rough surfaces are replaced with an equivalent rough surface in contact with a rigid flat surface [4]. The asperity profile $z(x)$ before deformation is:

$$z(x) = G^{D-1} l^{2-D} \cos\left(\frac{\pi x}{l}\right) \tag{1}$$

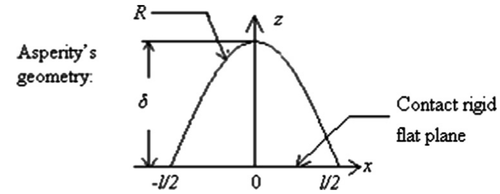

Fig. 1. The geometry of a contact spot of length scale l.

where D is the fractal dimension of the surface profile $(1<D<2)$, G is the fractal roughness parameter, and l is the length scale of a single asperity. In the MB model, the length scale l is the same as its real contact length. Hence, the asperity is fully deformed by the contacting plane, and the interference of any specific asperity is equal to its full height:

$$\omega = \delta = G^{D-1} l^{2(2-D)} \tag{2}$$

where ω is the interference of any specific asperity, and δ is the full height the asperity. The critical area of asperities is [9]:

$$a_{rc} = \frac{G^2}{(KH/2E)^{2/D-1}} \tag{3}$$

where H is the hardness of the material, E is the elastic modulus, and K is the hardness coefficient related to the Poisson ratio υ of the material by $K = 0.454 + 0.41\upsilon$. The critical area a_{rc} is a constant independent of the asperity's size. When $a_r < a_{rc}$, where a_r is the real contact area of asperities, the contact mode is plastic, and it becomes elastic when $a_r > a_{rc}$. In other words, an increasing contact area leads to a transition from the plastic to elastic contact mode. This unusual behavior contradicts classical contact mechanics and is unphysical. Such a shortcoming may affect many areas of tribology, such as friction, wear and adhesion. Unfortunately, many still continued to adopt the MB model concept despite its unphysical conclusion.

Morag and Etsion [29] have resolved the above mentioned drawback by offering a revision of the micro contact model of asperity. The ME model assumes that the deformation ω is an additional parameter independent of δ. The deformation ω can range from anywhere between zero to full deformation, i.e., $0 \le \omega \le \delta$. The asperity levels are denoted by the index n in the ME model:

$$l = 1/\gamma^n \tag{4}$$

where γ^n determines the frequency spectrum of the surface roughness $(\gamma > 1)$. The critical area a_{rc} is dependent on the scale l:

$$a_{rc} = \frac{1}{\pi}\left(\frac{\pi K H}{2E}\right)^2 \left(\frac{l^D}{G^{D-1}}\right)^2 \tag{5}$$

When $a_r < a_{rc}$, the contact mode is elastic, and it becomes plastic when $a_r > a_{rc}$. The contact mode transfers from elastic to plastic as the contact load and area increase, which is in accordance with classical contact mechanics and contrary to the unphysical result given by the MB model. In the ME model, the elastic contact load can be expressed by:

$$p_e = \frac{4\pi^{1/2} E G^{D-1}}{3 l_n^D} a_r^{3/2} \tag{6}$$

The relationship between the elastic contact area and contact load is of the form $a_r \propto p_e^{2/3}$, which is in agreement with the Hertz theory. The ME resolves the drawback in the MB model, but it does not provide a complete contact mode of the fractal rough surface.