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Algebraic approach to controller design using structured singular value

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ABSTRACT

The aim of this paper is to show an algebraic approach to controller design using the structured singular value (denoted μ) as a robust stability and performance indicator. The algebraic μ-synthesis is applied to three different problems–time-delay systems, the HIMAT vehicle model and the two-tank system. A way of treating general delayed systems with uncertain time delays via the linear fractional transformation is shown. A simple controller is derived, which handles uncertain time delay in both the numerator and denominator of an anisochronic system. The overall performance is verified by simulations for all systems and compared with the D–K iteration.

1. Introduction

The established algebraic theory (e.g. Kučera, 1972, 1993; Vidyasagar, 1985) is growing in importance due to the simplicity of controller derivation and the fact that some crucial properties of the resulting feedback loop can be easily influenced by the choice of the controller structure, which is conceivable within the scope of this approach. The structured singular value denoted μ (see Doyle, 1982, 1985) provides a measure of robust stability and performance that can take into account many aspects of controller design including sensor noise, dynamic perturbations as well as parametric uncertainties if the linear fractional transformation (LFT) can be used for its treatment. Standard tools for μ-synthesis provide a methodology for controller design with decoupling using model matching (see Prempain & Bergeron, 1998). However, the need of defining the internal model brings another step into the process of obtaining the controller. The algebraic approach provides the methodology for synthesis of very simple controllers (PI, PID) without an additional model matching design, which simplifies controller derivation.

Due to the multimodality of the cost function, an algorithm for global optimization is employed for tuning nominal closed-loop pole placement, where the peak of the μ-function in frequency domain gives the measure of controller stability and performance. Knowing this, behavior of the resulting feedback system can be simulated for the worst-case perturbation causing the highest value of μ. As a consequence, in case of time-delay systems it is possible to design controllers that have some specific properties such as stability and performance for the whole range of time delays. This implies that in practical settings resulting feedback loop characteristics will not degrade if time delay varies from 0 to a value defined as the worst possible case.

Many procedures have been developed for control of time-delay systems including LFT approaches using multiplicative uncertainty or IMC dealing with design in the ring of retarded quasipolynomial (RQ) meromorphic functions (e.g. Zítek & Hlava, 2001; Zítek & Kučera, 2003). The methods handling time-delay systems via multiplicative uncertainty are well known. However, the techniques for systems with time delay in both the numerator and denominator use mainly IMC design, which treats robustness in a more complex way.

Algebraic methods are easy to apply to SISO (single-input single-output) systems described by continuous or discrete transfer functions. Still, if used for MIMO (multi-input multi-output) systems, computational difficulties are severe. In this paper, the problem of the MIMO system design is solved via decoupling a MIMO system into two identical SISO plants, which are then approximated by transfer functions with a simple structure. This guarantees decoupled control and simplifies derivation of the pole placement formulae.

The algebraic μ-synthesis (Dlapa & Prokop, 2003) overcomes some difficulties connected with the D–K iteration, namely the fact that it does not guarantee convergence to a global or even local minimum (Stein & Doyle, 1991). Controllers obtained via the algebraic

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approach can have simpler structure due to the fact that there is no need of scaling matrices absorbance into generalized plant, and hence no need of further simplification causing deterioration of the frequency properties of the resulting controller. Moreover, the controller structure can be chosen in advance, which is not possible in the scope of currently used methods.

In this paper, the algebraic μ-synthesis is applied to the control of longitudinal model of a scaled remotely controlled version of the advanced fighter HIMAT and the two-tank system—classical examples of robust control design. The *D*–*K* iteration is used as a reference method and the results are compared through simulations for the nominal and perturbed plants.

Besides this, a general scheme for treating anisochronic delayed systems via LFT will be described together with an example of applications to such a system with time delays in both the numerator and denominator. Controller design using the algebraic μ-synthesis is presented alongside a comparative study with a standard tool. The overall performance is verified by simulations of step response for different values of time delays with simple feedback loop and two-degree-of-freedom structure (1DOF, 2DOF, see Prokop & Corriou, 1997).

The following notation is used: $\|\cdot\|_\infty$ denotes $\mathbf{H}_\infty$ norm, $\mathbf{R}$ and $\mathbf{C}^{n\times m}$ are real numbers and complex matrices, respectively, $\mathbf{I}_n$ is the unit matrix of dimension n and $\mathbf{R}_{PS}$ denotes the ring of Hurwitz-stable and proper rational functions.

2. Preliminaries

Define $\mathbf{\Delta}$ as a set of block diagonal matrices

$$\mathbf{\Delta} \equiv \{\mathrm{diag}[\delta_1 I_{r_1}, \ldots, \delta_S I_{r_S}, \Delta_1, \ldots, \Delta_F] : \delta_i \in \mathbf{C}, \Delta_j \in \mathbf{C}^{m_j \times m_j}\} \tag{1}$$

where S is the number of repeated scalar blocks, F is the number of full blocks, $r_1,\ldots,r_S$ and $m_1,\ldots,m_F$ are positive integers defining dimensions of scalar and full blocks.

For consistency among all the dimensions, the following condition must be held

$$\sum_{i=1}^{S} r_i + \sum_{j=1}^{F} m_j = n \tag{2}$$

Definition 1. For $\mathbf{M} \in \mathbf{C}^{n\times n}$ is $\mu_{\mathbf{\Delta}}(\mathbf{M})$ defined as

$$\mu_{\mathbf{\Delta}}(\mathbf{M}) \equiv \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(\mathbf{I} - \mathbf{M}\Delta) = 0\}} \tag{3}$$

If no such $\Delta \in \mathbf{\Delta}$ exists making $\mathbf{I} - \mathbf{M}\Delta$ singular, then $\mu_{\mathbf{\Delta}}(\mathbf{M})=0$.

Consider a complex matrix $\mathbf{M}$ partitioned as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \tag{4}$$

and suppose there is a defined block structure Δ_2 which is compatible in size with $\mathbf{M}_{22}$ (for any $\Delta_2 \in \mathbf{\Delta}_2$, $\mathbf{M}_{22}\Delta_2$ is square). For $\Delta_2 \in \mathbf{\Delta}_2$, consider the following loop equations

$$e = \mathbf{M}_{11} d + \mathbf{M}_{12} w$$

$$z = \mathbf{M}_{21} d + \mathbf{M}_{22} w$$

$$w = \Delta_2 z \tag{5}$$

If the inverse to $\mathbf{I} - \mathbf{M}_{22}\Delta_2$ exists, then e and d must satisfy $e=\mathbf{F}_L(\mathbf{M},\Delta_2)d$, where

$$\mathbf{F}_L(\mathbf{M}, \Delta_2) = \mathbf{M}_{11} + \mathbf{M}_{12}\Delta_2(\mathbf{I} - \mathbf{M}_{22}\Delta_2)^{-1}\mathbf{M}_{21} \tag{6}$$

is a linear fractional transformation on $\mathbf{M}$ by Δ_2, and in a feedback diagram appears as the loop in Fig. 1.

The subscript L on $\mathbf{F}_L$ pertains to the *lower* loop of $\mathbf{M}$ and is closed by Δ_2. An analogous formula describes $\mathbf{F}_U(\mathbf{M},\Delta_1)$, which is the resulting matrix obtained by closing the *upper* loop of $\mathbf{M}$ with a matrix $\Delta_1 \in \Delta_1$.

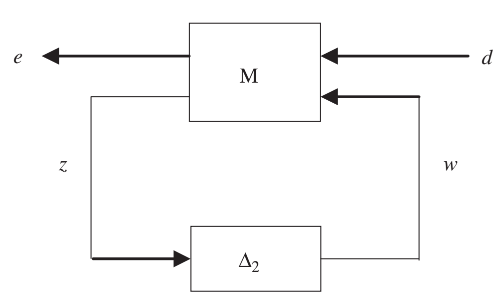

Fig. 1. LFT interconnection.