Contents lists available at ScienceDirect

Chemical Engineering Research and Design

journal homepage: www.elsevier.com/locate/cherd

An iterative LMI approach for H_∞ synthesis of multivariable PI/PD controllers for stable and unstable processes

Farshad Merrikh-Bayat

Department of Electrical and Computer Engineering, University of Zanjan, Zanjan, Iran

ARTICLE INFO

Article history:
Received 20 October 2017
Received in revised form 4 January 2018
Accepted 4 February 2018
Available online 14 February 2018

Keywords:
Bounded real lemma
H_∞ control
PID tuning
Multivariable process
Multi-input multi-output

ABSTRACT

An iterative linear matrix inequality (LMI) approach for designing multi-input multi-output (MIMO) PI/PD controller for stable/unstable multivariable processes is proposed in this paper. For this purpose, the matrix gains of controller are calculated such that the closed-loop system be stable, and simultaneously, the infinity norm of the weighted sensitivity function is minimized. This problem is mathematically formulated using the well-known bounded real lemma (BRL). The matrix inequality of the BRL is nonlinear because of multiplication of the variable of Lyapunov equation and gains of controller. To remove this nonlinearity, first a solution to the Lyapunov LMI is calculated using some necessary-type LMIs developed for this purpose. Then, this solution is substituted in the BRL to arrive at an LMI whose solution determines the gains of a stabilizing MIMO PI/PD controller which also minimizes the infinity norm of the weighted sensitivity function. If the resulting controller was not satisfactory, one can use the proposed iterative algorithm to improve its performance. The proposed method is used for tuning MIMO PI/PD for four stable/unstable MIMO processes.

1. Introduction

PIDs are low-order controllers which have been successfully used for decades to control higher-order processes. The classical single-input single-output (SISO) PID has three tuning parameters called proportional, integral and derivative gains. Since the number of tuning parameters is limited to three in SISO case, the problem of designing a SISO PID is rather simple and a wide variety of methods are available for this purpose. Some of the recently developed methods for SISO PID tuning are IMC-based PID tuning (Kumar and Sree, 2016), combination of IMC and model matching approach for closed-loop shaping (Jin and Liu, 2014), frequency loop-shaping (Grassi et al., 2001), generalized Kalman–Yakubovich–Popov (KYP) synthesis (Hara et al., 2006), optimization via multiobjective performance criterion (Sahib and Ahmed, 2016), and discrete-time fractional-order PID tuning based on meta-heuristic optimization (Merrikh-Bayat et al., 2015). Also, tuning SISO PID and fractional-order PID via LMI approach based on the frequency sampling method are discussed in Najafizadegan et al. (2017) and Merrikh-Bayat (2017), respectively, and SISO PID tuning based on convex–concave optimization is studied in Hast (Åström et al., 2013). See also Vilanova and Visioli (2006) and Vilanova and Visioli (2012) for some classical SISO PID tuning methods.

Multi-input multi-output (MIMO) PID controllers also had been the subject of many studies. For example, designing multi-loop PID controllers (Huang and Huang, 2004; Wang et al., 2007), equivalent transfer function method for PI/PID controller design (Xiong et al., 2007), the method of complex/real ratio of the characteristic matrix eigenvalues (Ruiz-López et al., 2006) and an analytical method for stabilizing MIMO PID synthesis (Gündeş et al., 2007) can be found in the literature. A summary of some classical methods for designing MIMO PID can also be found in Wang et al. (2008). It should be noted that the problem of designing a MIMO PID is intrinsically more complicated than a SISO PID mainly because in MIMO case the number of variables is drastically increased by increasing the number of inputs and outputs of

E-mail address: f.bayat@znu.ac.ir
https://doi.org/10.1016/j.cherd.2018.02.012

the process. For example, a standard MIMO PID used to control a process with 3 inputs and 3 outputs has 27 tuning parameters, which makes the problem more challenging compared to SISO case.

Although PIDs constitute more than 90% of the controllers used in industry, there are still many problems where other controllers lead to considerably better results (Vilanova and Visioli, 2006). It justifies the continuous effort, especially in the field of MIMO PID, to develop more effective tuning methods for designing higher performance PIDs. Linear matrix inequalities (LMIs) are used for this purpose in the past years; see for example (Pradhan and Ghosh, 2015; Boyd et al., 2016; Wu et al., 2011; Zheng et al., 2002; Lin et al., 2004; Ge et al., 2002). The big advantage of LMIs is that they are convex, and consequently, can be solved very effectively by using algorithms like the interior point method in polynomial time. Although many problems in the field of control theory like stability analysis (Sabatier et al., 2010), calculating the H_∞ norm of a linear system transfer function (Skogestad and Postlethwaite, 2005, Ch. 12), calculating the upper bound on μ (Skogestad and Postlethwaite, 2005, Ch. 12) and state-feedback control (Farges et al., 2010; Balochian et al., 2011) can be formulated using LMIs, many others are non-convex and cannot be represented by LMIs. A large effort is made to overcome the non-convexity of such problems and finding (approximate) solutions using LMIs. Two well-known methods to transform non-convex and bilinear matrix inequalities (BMIs) to LMIs are convex–concave decomposition and linearization method (Tran Dinh et al., 2012).

To the best knowledge of author, Zheng et al. (2002) is the first considerable work on tuning MIMO PIDs via LMI approach. The basic idea used in Zheng et al. (2002) is to transform the problem of designing a MIMO PID to a static output feedback whose solution via LMI approach was already known. The method presented in Lin et al. (2004) for MIMO PID tuning is intrinsically the same as Zheng et al. (2002) but some of the restrictions of Zheng et al. (2002) are removed. Another considerable work in the field of MIMO PID tuning via LMI approach is Boyd et al. (2016). In that paper the gains of multivariable PID are calculated by minimizing the low frequency gain of open-loop system subject to constraints on infinity norms of closed-loop transfer functions. The problem under consideration in Boyd et al. (2016) is non-convex and approximate solutions are obtained by combining a matrix extension of convex–concave decomposition and the frequency sampling method. One feature of Boyd et al. (2016), Zheng et al. (2002) and Lin et al. (2004) is that they are of iterative nature which means that an initial point is required to begin the search. For example, in Boyd et al. (2016) the search for the (sub)optimal solution should begin from a stabilizing solution, which limits the application of that method. Moreover, the methods presented in Boyd et al. (2016), Zheng et al. (2002) and Lin et al. (2004) work based on some sufficient-type conditions where finding a solution (if one exists) is not guaranteed. On the other hand, since the method presented in Boyd et al. (2016) does not rely on state-space equations, it can be applied directly to systems with time delay.

In the method proposed in this paper for tuning MIMO PI/PD via LMI approach two sets of LMIs are generated and solved. First, a solution to the nonlinear Lyapunov equation (to achieve closed-loop stability at the presence of a MIMO PI/PD with unknown parameters in the loop) is obtained based on some necessary-type LMIs developed for this purpose. Then the resulting solution is employed in the bounded-real

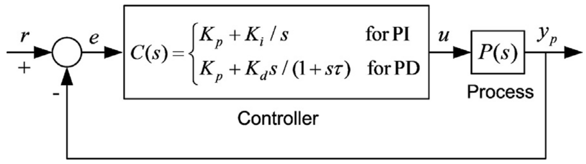

Fig. 1 – The feedback system with MIMO PI/PD controller.

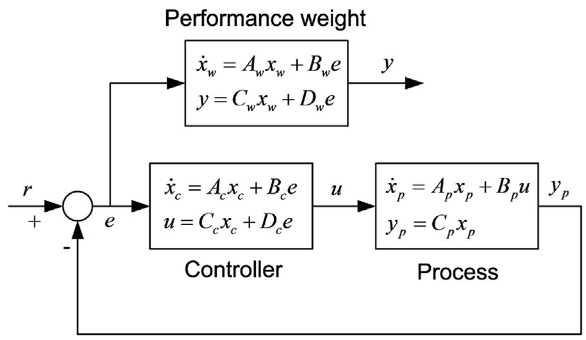

Fig. 2 – Representing the MIMO PI/PD tuning as a weighted sensitivity problem.

lemma (BRL) to calculate the parameters of controller such that the singular values (SVs) of the sensitivity function are well shaped. If the resulting controller is not satisfactory, the proposed iterative approach can be used to enhance its performance.

The propounded method has some advantages and disadvantages compared to the existing methods for tuning MIMO PID via LMI approach. One advantage of the proposed method is that no initial point is required to begin the search for the gains of controller. Moreover, it can also be applied to both stable and unstable processes. The main limitation of the proposed method is that it is based on necessary-type LMIs for closed-loop stability. Hence, similar to Boyd et al. (2016), Zheng et al. (2002) and Lin et al. (2004) finding a solution is not guaranteed. However, the simulation results show that the proposed necessary LMIs for closed-loop stability very often lead to a controller with desired performance.

The rest of this paper is organized as follows. The main results, including the proposed algorithm for tuning MIMO PI/PD, is presented in Section 2. Four illustrative examples are presented in Sections 3 and 4 concludes the paper.

2. Main results

2.1. *Problem description*

Our aim here is to design the stabilizing multivariable PI/PD controller C(s) in Fig. 1 which minimizes $\|W_p(s)S(s)\|_\infty$ where $W_p(s)$ is a (stable and scaler) performance weight, $r, e, y_p \in \mathbb{R}^{l\times 1}$, $u \in \mathbb{R}^{m\times 1}$, $K_p, K_i, K_d \in \mathbb{R}^{m\times l}$ are unknown controller gains, $\tau > 0$ is the predetermined time-constant of derivative filter, and $S(s) := (I + PC)^{-1}PC$ is the sensitivity function. Without a considerable loss of generality, P(s) is assumed to be a strictly proper transfer function matrix, that is $\lim_{s\to\infty} P(s) = 0$.

This problem is equivalent to calculating the unknown matrices of state-space realization of controller in Fig. 2 such that firstly the closed-loop system be internally stable, and secondly the infinity norm of the transfer function matrix from r to y is minimized. Note that although the problem is originally defined in the frequency domain, it will be for-