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Maximum sensitivity based analytical tuning rules for PID controllers for unstable dead time processes

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ABSTRACT

In this paper, maximum sensitivity and internal model control (IMC) based proportional-integral-derivative (PID) controllers are designed for unstable first-order plus-dead-time (UFOPDT) processes. The designed controller parameters are functions of the UFOPDT model parameters and the IMC closed loop tuning parameter. The tuning parameter plays a vital role and determines the closed loop performance and robustness of the designed controller. Systematic guidelines are provided for selection of this tuning parameter based on maximum sensitivity. Analytical tuning rules are developed for the controller parameters for different time delay to time constant ratios with desired level of robustness. These controller settings allow the operator to deal with the closed-loop control system performance-robustness trade-off by specifying the robustness level (maximum sensitivity). Simulation studies have been carried out on various UFOPDT processes to explain the advantages of the proposed analysis.

1. Introduction

Open loop unstable processes arise frequently in the chemical (distillation, polymerization reactors, heat exchangers, etc.) and biological (fermentation) processes and are fundamentally difficult to control than that of the stable processes and the difficulty increases when there exist a time delay. Time delays occur frequently in process control problems, because of the distance velocity lags, recycle loops, and composition analysis loops. The performance specifications that are usually achieved for stable systems are difficult to achieve for unstable systems. The closed loop response for such processes exhibits large overshoots and settling times. As has been widely reported, PID controllers are with no doubt, the controllers most extensively used in the process industry.

The PID controller design methods for unstable processes have been addressed by many researchers (Rao and Chidambaram, 2006, 2012). Sree et al. (2004) and Sree and Chidambaram (2006) described occurrence and existence of unstable systems in engineering processes and provided an excellent overview of controller design techniques for unstable processes. They developed tuning rules based on equating coefficient method for first order unstable processes with time delay. Arrieta et al. (2011) proposed PID tuning based on servo or regulatory operations. Nasution et al. (2011) proposed PID controller design based on IMC method and H_2 minimization and Maclaurin series approximation and obtained improved performances over many previous methods. They have developed five different desired closed loop transfer functions and designed the controllers correspondingly based on these five desired closed loop transfer functions and recommended one. Shamsuzzoha and Lee (2008) showed that PID controllers in series with lead/lag compensators provide improved closed loop performances when compared to that of PID controller alone for UFOPDT processes. Further, complex control structures have also been developed such as modified Smith predictor (Rao et al., 2007; Uma et al., 2010; Normey-Rico and Camacho, 2008), modified IMC (Tan et al., 2003) and

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two-degrees-of-freedom control structures (Liu et al., 2005; Tan, 2010) with more number of controllers and also with more complexity in the design of controllers for UFOPDT processes to improve the closed loop performance. In most of these controllers, there exists performance-robustness trade-off and the tuning parameter should be selected in such a way that the resulting controllers give both nominal as well as robust performance as a function of peak value of the sensitivity function i.e. maximum sensitivity (Ms). Moreover, PID controller cannot provide stabilized responses when the time delay to time constant ratio is greater than 1.2 for unstable systems. It can be observed that complex control schemes such as modified Smith predictor and two degrees of freedom structures where there are more than two controllers involved are not desirable for practical purposes and these modified schemes also are not applicable when the time delay to time constant ratio (T_o) exceeds 1.2. Hence, keeping the simplicity into account, properly designed PID controller is better than these modified schemes. However, the designed PID controller should provide good nominal and robust closed loop responses and smooth manipulated variable responses. Alfaro et al. (2010) have developed analytical equations for PI controller for stable systems based on Ms. Arrieta and Vilanova (2012) developed PID tuning rules for stable systems for servo/regulatory problems based on Ms. They formulated an optimization problem with constraints to design the controller. However, such rules do not exist for unstable systems.

Based on this motivation, in the present work, an attempt is made to develop analytical tuning rules for PID controllers as a function of Ms for unstable processes using IMC–H_2 minimization theory. This approach explicitly considers the control system performance-robustness trade-off aiming to obtain a smooth response to both disturbance and set point step changes and at the same time to guarantee a minimum robustness level. The distinctive feature of the resulting tuning procedure is that the designer may select one of four different robustness levels in the range $2 \le Ms \le 10.5$ for UFOPDT process models with time delay to time constant ratio (T_o) in the range $0.10 \le T_o \le 0.9$. For clear illustration, the paper is organized as follows. Controller design is addressed in Section 2 followed by the proposed Ms based tuning rules in Section 3. Simulation studies are explained in Section 4 followed by discussion in Section 5 and finally conclusions are presented in Section 6.

2. Controller design

The closed-loop control structure of IMC is shown in Fig. 1, where $G_p(s)$ is the transfer function of the unstable process, $G_m(s)$ is the corresponding transfer function model and Q_C is the transfer function of the IMC controller. Here, the controller design is addressed for UFOPDT processes. Recently, Vanavil et al. (2014) have developed analytical tuning formulas for PID controller with a lead-lag filter for UFOPDT processes. Their method is briefly discussed here. The UFOPDT process transfer function is considered as

$$G_p(s) = \frac{k_P e^{-\theta s}}{\tau_P s - 1} \tag{1}$$

According to IMC principle, the IMC controller Q_C is equivalent to

$$Q_C = \tilde{Q}_C F \tag{2}$$

where F is a filter which is used for altering the robustness of the controller. The filter structure should be selected such that the IMC controller Q_C is proper and realizable and also the control structure is internally stable. In addition to these requirements, it should be selected such that the resulting controller provides improved closed loop performances. $\tilde{Q}_C$ is designed for a specific type of step input disturbance (υ) to obtain H_2 optimal performance (Morari and Zafiriou, 1989) and is based on the invertible portion of the process model. The process model and the input are divided as

$$G_m = G_{m-}G_{m+} \quad \text{and} \quad \upsilon = \upsilon_-\upsilon_+ \tag{3}$$

where the subscript "−" refers to minimum phase part and "+" refers to non-minimum phase part. The Blaschke product of RHP poles of G_m and υ are defined as

$$b_m = \prod_{i=1}^{k} \frac{-s + p_i}{s + \bar{p}_i} \quad \text{and} \quad b_\upsilon = \prod_{i=1}^{\tilde{k}} \frac{-s + p_i}{s + \bar{p}_i} \tag{4}$$

where p_i and $\bar{p}_i$ are the ith RHP pole and its conjugate respectively. Based on this, the H_2 optimal controller is derived by using the following formula given by Morari and Zafiriou (1989).

$$\tilde{Q}_C = b_m(G_{m-}b_\upsilon\upsilon_-)^{-1}\{(b_m G_{m+})^{-1} b_\upsilon \upsilon_-\}|_* \tag{5}$$

where $\{\ldots\}|_*$ is defined as the operator that operates by omitting all terms involving the poles of $(G_{m+})^{-1}$ after taking the partial fraction expansion. This idea is applied successfully by Nasution et al. (2011) and derived IMC based PID controller. The same derivation for obtaining the IMC controller $\tilde{Q}_C$ for UFOPDT processes is given here for clear understanding. Considering perfect model case i.e. $G_p = G_m$, first, split the process model and input into minimum and non-minimum phase parts as

$$G_{m-} = \frac{-k_P}{\tau_P(-s + (1/\tau_P))} \quad \text{and} \quad G_{m+} = e^{-\theta s} \tag{6}$$

$$\upsilon_- = \frac{-k_P}{\tau_P(-s + (1/\tau_P))s} \quad \text{and} \quad \upsilon_+ = 1 \tag{7}$$

Then, the Blaschke product is obtained as

$$b_m = \frac{(-s + (1/\tau_P))}{(s + (1/\tau_P))} \quad \text{and} \quad b_\upsilon = \frac{-s + (1/\tau_P)}{s + (1/\tau_P)} \tag{8}$$

Substituting all expressions in Eq. (5), one will get,

$$\tilde{Q}_C = \frac{(\tau_P s - 1)}{k_P}\{(e^{\theta/\tau_P} - 1)\tau_P s + 1\} \tag{9}$$

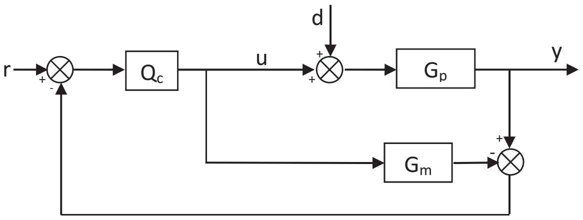

Fig. 1 – IMC control scheme.