Desalination 433 (2018) 151–154

Contents lists available at ScienceDirect

Desalination

journal homepage: www.elsevier.com/locate/desal

Marangoni stress induced by free-surface for pressure reduction in reverse osmosis

Francisco J. Arias

Department of Fluid Mechanics, University of Catalonia, ESEIAAT C/ Colom 11, Barcelona 08222, Spain

ARTICLE INFO

Keywords:
Reverse osmosis
Marangoni stress
Concentration gradient

ABSTRACT

Marangoni hydrodynamic motion and its potential technological application in reverse osmosis (RO) process for seawater desalination is discussed. The fundamental core idea in this note is the possibility to take advantage of the inherent concentration gradient in a RO process. It is well known that to run a RO process, it is necessary to apply a hydrodynamic pressure to overcome the osmotic pressure, however, by inducing a free-surface, e.g., a Leidenfrost surface, on the membrane wall, an additional hydrodynamic Marangoni stress could be generated, which, likewise than the osmotic pressure is driven by the concentration gradient but acting in the opposite direction, i.e., reducing the external hydraulic pressure to be applied. Utilizing a simplified geometrical and physical model, an analytical expression for the pressure reduction was derived. One important preliminary result in this work, is that the Marangoni stress can provide pressure against the osmotic pressure for membrane porous that are less than micrometric size.

1. Introduction

Reverse osmosis (RO) encompasses several of the most important filtration process for industrial exploitation, namely: microfiltration; ultrafiltration; and nanofiltration. In short, because molecules naturally move from areas of high concentration to low concentration, then if it is desired that molecules move from areas of low concentration to high concentration it is necessary to apply an external pressure. The pressure difference on both sides of the membrane will cause the permeate to cross the membrane at a steady state, hence, the name pressure-driven operation.

Whereas RO process is one of the most researched fields in membrane technology and embracing many topics, e.g., optimization, new materials, performance; [1–16], just to name a few, nevertheless, there are still aspects which deserve an attack from a theoretical point of view.

The object of this note was a first preliminary investigation on the possibility of Marangoni induced stress (by inducing a free-surface) and its use with regard to pressure-driven membrane filtration technology. The justification behind the idea is straightforward: the salinity gradient which is causing the osmosis pressure which must be overcome to run the filtration process by applying an external hydrodynamic pressure, can actually generate an opposite force (Marangoni stress) which can counteract the osmotic pressure and then reducing the effective applied external pressure which translates into a reduction of cost of the filtration process.

2. Statement of the core idea

2.1. Momentum considerations

For the sake of generality, we will consider a simple problem of motion of an incompressible viscous fluid with a certain concentration. Let the fluid be enclosed between two parallel walls (top and bottom walls of a rectangular membrane). Also, in the top wall a free-surface is induced. Such a free-surface could be induced by, say, a Leidenfrost surface created by heating the surface beyond its critical heat flux CHF, or by injecting gas. Suffice to say, that generating a sustained Leidenfrost surface for water or similar liquids does not requires substantial power input if one takes into account the length scale of the porous, e.g., for the case of water the critical heat flux (CHF) is approximately 1000 kW/m^2 and if one considers porous sizes of micrometric size or even smaller, then the power needed per porous will be very low. Fig. 1 is a sketch of the problem considered.

Now, if it is desired to run a pressure-driven filtration operation, i.e., the separation of solvent from the highly concentrated solution (left side in Fig. 1) and going to the low concentration side (right side in the same figure), in order to do this, it is necessary to apply an external hydraulic pressure, let us call this as Δp. This pressure must be able to sustain the flow against the hydrodynamic forces opposing this motion namely, skin drag friction and the osmotic pressure π. However, in addition, because of the induced free-surface, we have an additional force at the free-surface or Marangoni stress which is favoring the Ro-

E-mail address: francisco.javier.arias@upc.edu.

https://doi.org/10.1016/j.desal.2018.01.006
Received 5 June 2017; Received in revised form 30 August 2017; Accepted 4 January 2018

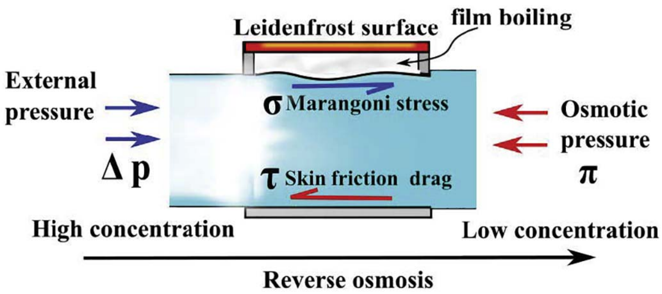

Fig. 1. Physical model for the two parallel planes with a free surface at the top of the channel.

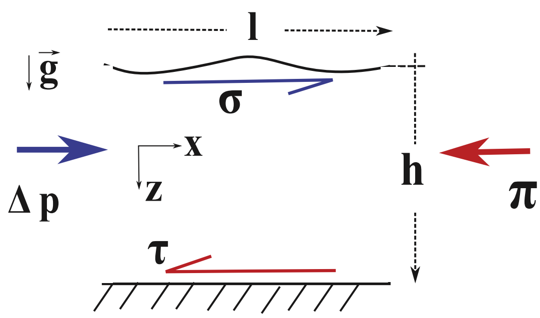

Fig. 2. Schematics physical model for the two parallel planes with free surface at the top.

filtration, i.e., pushing the liquid from the high concentration towards the low concentration. From this simple generalized picture, we can start our momentum considerations.

To begin with, the Navier-Stokes equations for the system schematically depicted in Figs. 1 and 2 is reduced to the following set of equations, [17]

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} \tag{1}$$

and

$$\frac{\partial p}{\partial z} = \rho \mathbf{g} \tag{2}$$

where u is the velocity of the fluid ($u = u(x)$); z is the vertical axis; x the longitudinal-axis; p is the pressure; ρ the density of the fluid; μ the dynamic viscosity; and $\mathbf{g}$ the gravity.

If we express the pressure as

$$p = \rho \mathbf{g} z + p_x(x) \tag{3}$$

then Eq. (1) can be rewritten as

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{dp_x(x)}{dx} \tag{4}$$

since $p_x(x)$ does not depend on z coordinate.

Solving Eq. (4), we obtain

$$u = \frac{1}{2\mu}\frac{dp_x(x)}{dx}z^2 + c_1 z + c_2 \tag{5}$$

where c_1 and c_2 are constants to be determined by proper boundary conditions.

The boundary conditions are

$$u(z = h) = 0 \tag{6}$$

and the Marangoni stress at the free-surface $z = 0$ results

$$\left.\frac{\partial \gamma}{\partial x}\right|_{z=0} = -\mu \left.\frac{\partial u}{\partial z}\right|_{z=0} \tag{7}$$

where γ is the surface tension of the liquid. By applying those boundary conditions, we obtain

$$c_1 = -\frac{1}{\mu}\frac{d\gamma}{dx} \quad ; \qquad c_2 = \frac{h}{\mu}\frac{d\gamma}{dx} - \frac{1}{2\mu}\frac{dp_x(x)}{dx}h^2 \tag{8}$$

which inserted into Eq. (5) yields

$$u = \frac{1}{\mu}\frac{d\gamma}{dx}(h - z) - \frac{1}{2\mu}\frac{dp_x(x)}{dx}(h^2 - z^2). \tag{9}$$

Finally, the pressure drop along the plate can be calculated from the mass fluid velocity (averaged over the depth of the liquid) as

$$\bar{u} = \frac{1}{h}\int_0^h u dz \quad ; \tag{10}$$

and after integration one obtains

$$\frac{dp_x(x)}{dx} = \frac{3}{2h}\frac{d\gamma}{dx} - \frac{3\mu}{h^2}\bar{u}. \tag{11}$$

Taking into account that in our case the surface tension gradient is driven by a concentration gradient, therefore, Eq. (11) can be rewritten as

$$\frac{dp_x(x)}{dx} = \frac{3}{2h}\frac{d\gamma}{dc}\frac{dc}{dx} - \frac{3\mu}{h^2}\bar{u} \tag{12}$$

where c is concentration. From Eq. (1), because u is only function of z and $p_x(x)$ of x then we deduce that $\frac{dp_x(x)}{dx} = constant$; then, the pressure gradient may be written as $-\frac{\Delta p}{l}$, where l is the length of the channel, and Δp is the positive pressure drop (the inlet pressure minus the outlet pressure). Thus, Eq. (12) becomes

$$\Delta p \approx \frac{3}{2h}\frac{d\gamma}{dc}\Delta c + \frac{3\mu l}{h^2}\bar{u} \tag{13}$$

where $\frac{dc}{dx} \approx -\frac{\Delta c}{l}$, being Δc the positive change in concentration (the high concentration minus the low concentration), and where if $\Delta c = 0$ — or not free-surface exist, we recover, of course, the expression for a fluid enclosed between two parallel planes, [17].

Finally, we need to add the additional external hydraulic pressure needed to overcome the resulting osmotic pressure $\Delta\pi$ between both sides, and then, the final expression for pressure drop is given by

$$\Delta p \approx \frac{3}{2h}\frac{d\gamma}{dc}\Delta c + \Delta\pi + \frac{3\mu l}{h^2}\bar{u}. \tag{14}$$

The above expression may be further simplified by taking into account that the osmotic pressure is dependent — as the Marangoni term, of the concentration gradient. Indeed, the osmotic pressure can be expressed as a function of a certain pressure taken as reference, let us call π_o the osmotic pressure at a given reference concentration, say c_o, and then the osmotic pressure can be written as

$$\Delta\pi = \frac{\pi_o}{c_o}\Delta c \tag{15}$$

and then, Eq. (14) can be expressed as

$$\Delta p = \underbrace{\left(\frac{\pi_o}{c_o} + \frac{3}{2h}\frac{d\gamma}{dc}\right)\Delta c}_{concetration} + \underbrace{\frac{3\mu l}{h^2}\bar{u}}_{drag} \tag{16}$$

because $\frac{d\gamma}{dc} < 0$, it is instructive using its absolute value, i.e., $\frac{d\gamma}{dc} = -\left|\frac{d\gamma}{dc}\right|$, and then

$$\Delta p = \underbrace{\left(\frac{\pi_o}{c_o} - \frac{3}{2h}\left|\frac{d\gamma}{dc}\right|\right)\Delta c}_{concetration} + \underbrace{\frac{3\mu l}{h^2}\bar{u}}_{drag}. \tag{17}$$

The above expression is showing clearly that Marangoni stress is acting in the opposite direction than osmotic pressure and reducing the