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Hybrid real–complex current injection-based load flow formulation

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ABSTRACT

This paper presents a new current injection-based load flow procedure in which the Y-bus matrix becomes the main building block of the Jacobian. Starting from a conventional complex formulation, an augmented incremental model is developed comprising both complex and real unknowns, corresponding to PQ and voltage-regulated buses respectively. Then, details are provided on the incorporation of voltage regulating devices (PV buses and tap changers) into the proposed hybrid model. Test results are presented showing that, when solution adjustments are considered, the proposed approach is competitive with the conventional power mismatch-based Newton's method, and performs much better than sensitivity-based schemes usually adopted in combination with current injection-based load flow solutions.

1. Introduction

Since the pioneering work of Tinney and others [1,2] there have been uninterrupted efforts to develop improved load flow (LF) solution methods and related artifacts, such as the inclusion of voltage regulating devices. The reader is referred to the excellent survey [3] or the more recent compilation provided in [4] (Chapter 3) for a detailed account of the power flow problem.

Load flow solution methods can be broadly classified in three main categories: (1) General-purpose solution based on the application of the Newton-Raphon (NR) iterative scheme to the nonlinear power mismatch equations [1]; (2) application of the NR scheme to the current mismatch equations [5–7]; these methods are particularly attractive in the three-phase case [8,9] or in harmonic load flow solutions [10,11]; (3) for strictly radial systems, both single- and three-phase ones, it is possible to apply any of the so-called forward/backward sweep methods [12,13]; these last methods can be extended to weakly meshed systems by compensating the radial solution [14,15], but then their original simplicity is somewhat lost and the number of iterations tends to increase dramatically.

Load flow tools must face the need to model the action of local controllers, such as automatic voltage regulators associated with generators, shunt devices or on-load tap changers (OLTC). In its simplest form, a local controller tries to keep a controlled quantity as close as possible to a target value, by acting on the corresponding control variable within its physical limits (more involved control schemes, including deadband, hysteresis, etc. are out of the scope of this paper).

In the load flow solution process this can be accomplished in several ways, which can be broadly grouped in two major categories: a first one where both the state and mismatch vectors are dynamically adapted at each iteration to account for the constraints imposed by controlling devices, usually leading to an augmented Jacobian. This scheme is adopted for instance to accommodate on-load tap changers or phase shifters in the NR method [16], but also when handling PV buses in rectangular coordinates [17,18]. The inclusion of PV buses in the polar formulation is a notable exception, since a model reduction arises by removing the controlled quantity from the state vector [1]. A second category arises if each NR iteration is performed for given control settings. Then, control variables are adjusted for the next iteration in proportion to the residual of the respective regulated quantities, with the help of pre-computed sensitivities or by directly using the actual gains of the local controllers being emulated (seldom available to the load flow user). This last approach is inherently associated with methods using constant Jacobians [19,20], but is also common in distribution load flows [21]. As pre-computed sensitivities ignore mutual effects, this scheme may lead to unexpected oscillations and poor convergence [22], particularly in distribution systems where the interaction among nearby controllers is not negligible [23,24].

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As summarized above, polar coordinates are the preferred choice in the power-mismatch formulation, while current-injection methods have been systematically associated with rectangular coordinates. In this work, a more compact, mixed current-injection formulation is developed in which unknowns related to PQ buses appear in complex form (voltage phasors) whereas the necessary real unknowns (in polar coordinates) are retained for voltage-regulated buses. This way, the major diagonal block of the Jacobian, corresponding to PQ buses, is simply the respective Y_{bus} submatrix, which remains constant throughout the iterative process. The resulting procedure is general enough for any type of network, but can be of particular interest for high-voltage distribution systems, not necessarily radial, usually containing fewer control devices in relative terms than transmission systems.

The paper is organized as follows: after the introduction, the basic load flow formulation in complex form is reviewed in Section 2. Then, Section 3 introduces the mixed real–complex incremental model proposed in this work, including PV buses and tap changers, while Section 4 succinctly provides the solution procedure. Next, Section 5 compares the performance of the proposed method with other well-known procedures. Finally, Section 6 summarizes the main contributions of this work.

2. Load flow equations in complex form

This section is devoted to review the well-known load flow equations and to introduce the basic notation.

2.1. Nodal network equations

Let $\mathcal{U}_b$ and $\mathcal{I}_b$ denote the vectors of bus voltage and net injected current phasors, respectively. Then, the nodal equations can be written as follows:

$$\mathcal{Y}_b\mathcal{U}_b = \mathcal{I}_b \quad (1)$$

where $\mathcal{Y}_b$ is the so-called bus admittance matrix, which is symmetric in absence of phase shifting transformers. When load and generation buses are separately considered, the following partitioned form arises,

$$\mathcal{Y}_{ll}\mathcal{U}_l + \mathcal{Y}_{lg}\mathcal{U}_g + \mathcal{Y}_{lo}\mathcal{U}_o = \mathcal{I}_l + \mathcal{I}_{sl} \quad (2)$$

$$\mathcal{Y}_{gl}\mathcal{U}_l + \mathcal{Y}_{gg}\mathcal{U}_g + \mathcal{Y}_{go}\mathcal{U}_o = \mathcal{I}_g + \mathcal{I}_{sg} \quad (3)$$

where subscripts g and l refer respectively to generation and load buses and $\mathcal{U}_o$ is the slack bus voltage phasor. Notice that $\mathcal{Y}_{lo}$ and $\mathcal{Y}_{go}$ are extremely sparse column vectors (non-zero components correspond to buses directly linked to the slack bus).

In this work, OLTC transformers will be modeled through additional shunt currents, as explained below. Therefore, in the general case, the vector of bus injected currents can be split into the following two components:

- $\mathcal{I}_l$ and $\mathcal{I}_g$, representing the 'ordinary' currents injected by loads and generators.
- $\mathcal{I}_{sl}$ and $\mathcal{I}_{sg}$, which are the additional shunt currents injected by transformers with off-nominal tap value.

Both current components will be separately addressed in the sequel.

2.2. Bus constraints for loads and generators

Depending on the bus type, different nonlinear constraints apply to $\mathcal{I}_l$ and $\mathcal{I}_g$.

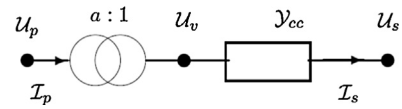

Fig. 1. Tap-changer transformer model.

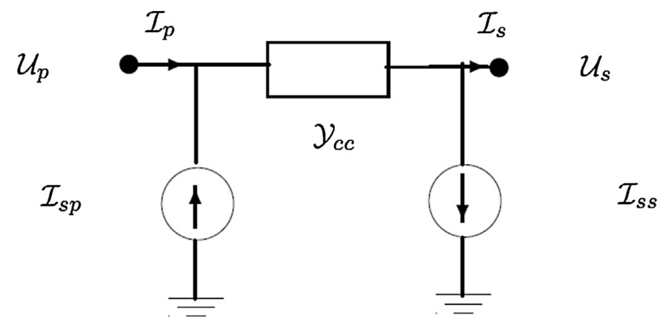

Fig. 2. Resulting tap-changer transformer model with constant series admittance.

B.1. Load or PQ buses

For each PQ bus the net complex power is specified while the voltage phasor is unknown. Let $\mathcal{S}_l^{sp} = P_l^{sp} + jQ_l^{sp}$ be a vector composed of specified complex powers for all PQ buses. Then, the following equation in matrix form applies:

$$\mathcal{S}_l^{sp} = P_l^{sp} + Q_l^{sp}j = \text{diag}(\mathcal{U}_l)\mathcal{I}_l^* \quad (4)$$

where $\text{diag}(\mathcal{U}_l)$ represents a diagonal matrix composed of voltage phasors for all PQ buses.

B.2. Generator or PV buses

The net active power P_g^{sp} and the voltage magnitude V_g^{sp} are specified at PV buses, whereas the voltage angle θ_g and injected reactive power Q_g are unknown quantities. In matrix form, for all PV buses,

$$P_g^{sp} + jQ_g = \text{diag}(\mathcal{U}_g)\mathcal{I}_g^* \quad (5)$$

with $|\mathcal{U}_g| = V_g^{sp}$.

By replacing $\mathcal{I}_l$ and $\mathcal{I}_g$ from (4) and (5) into (2) and (3), the nonlinear load flow equations of the conventional formulation are obtained, in complex form [4].

2.3. Additional shunt currents injected by tap changers

A generic tap-changer transformer (Fig. 1), can be modeled, as shown in Fig. 2, by a series admittance $\mathcal{Y}_{cc}$ and two equivalent shunt currents representing the effect of off-nominal tap values, given by:

$$\mathcal{I}_{sp} = \mathcal{Y}_{cc}\left[\frac{(a^2-1)}{a^2}\mathcal{U}_p - \frac{(a-1)}{a}\mathcal{U}_s\right] \quad (6)$$

$$\mathcal{I}_{ss} = \mathcal{Y}_{cc}\frac{(a-1)}{a}\mathcal{U}_p \quad (7)$$

where a is the tap value (in pu) and subscripts p and s denote the primary and secondary buses, respectively.

Notice that both $\mathcal{I}_{sp}$ and $\mathcal{I}_{ss}$ are null for $a = 1$. Therefore, vectors $\mathcal{I}_{sl}$ and $\mathcal{I}_{sg}$, representing in (2) and (3) the additional shunt currents injected by tap changers, will be null except for those entries corresponding to buses (p and s) which are incident to off-nominal tap changers.

3. Linearized equations in complex-real form

It is well known that partial derivatives cannot be computed in complex form in the presence of the complex conjugate operator. For this reason, the standard NR iterative method, requiring the computation of a Jacobian matrix, cannot be applied unless polar