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Multi-agent control of community and utility using Lagrangian relaxation based dual decomposition

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ABSTRACT

In multi-agent based demand response program, communities and a utility make decisions independently and they interact with each other with limited information sharing. This paper presents the design of multi-agent based demand response program while considering ac network constraints. This project develops two types of information sharing and iterative decision making procedures for the utility and communities to reach Nash equilibrium. The distributed algorithms of decision making are based on Lagrangian relaxation, duality, and the concept of upper and lower bounds. The first algorithm is subgradient iteration based distributed decision making algorithm and the second algorithm is based on lower bound and upper bound switching. The two algorithms require different information flow between the utility and communities. With the adoption of distributed algorithms, the utility solves optimal power flow at each iteration while considering ac network constraints, and the communities also conduct optimization. Through information sharing, the utility and the communities update their decisions until convergence is reached. The decision making algorithms are tested against three test cases: a distribution network IEEE 399 system, two meshed networks (IEEE 30-bus system and IEEE 300-bus system). Fast convergence is observed in all three cases, which indicates the feasibility of the demand response design.

1. Introduction

Multi-agent control has been applied in microgrid or demand side interactions with utility [1–4]. Microgrids or demand side make their own decisions while exchanging limited information with the grid. For example, in [1], an optimal demand response is designed for the demand sides to bid the amount of load shedding as a supply function of price. The utility collects the bids from all demand sides and update the price. In [2], the demand sides send the utility the information on their demands, and the utility sets the prices. The demand sides update their demand requests upon receiving the price.

In all above mentioned references, distribution networks are either represented in a simplified way or not represented at all. The objective of this paper is to implement multi-agent control of demand sides and utility while considering ac network constraints including line flow limits and bus voltage limits.

Implementing multi-agent control requires distributed algorithms. For optimization problems, there are ways to decompose and construct distributed optimization algorithms [5]. In the field of communication layering problems, primal decomposition, dual decomposition, and primal-dual decomposition can be applied in different scenarios [6]. In power systems optimization problems, due to the decoupled cost function structure and coupled constraints, Lagrangian relaxation based dual decomposition is commonly used. Example applications can be found in aggregated PHEV control considering global constraint [7], and distributed voltage control [8].

In optimization decomposition, an original problem is separated into a master problem along with many subproblems with small sizes. After each subproblem is solved, the main problem is solved adopting iterative methods such as subgradient update. Subgradient algorithm based on Lagrangian relaxation has been applied by Luh *et al* for manufacturing job scheduling [9]. Zero or small duality gap can prove that the solution is optimal or very close to optimal. In game theory, iteration means each agent in the system is exchanging information and learning to reach a Nash equilibrium.

Not all distributed algorithms have the information exchange structure suitable for multi-agent control [10]. In this research, distributed algorithm and learning methods suitable for multi-agent based microgrid and utility interaction will be examined.

Subgradient update based distributed algorithms are popular as seen in the literature. One shortcoming of subgradient method is its slow convergence speed. Scaling factors of Lagrangian multipliers need to be updated to enhance convergence. The update is dependent on specific problems. An improved Lagrangian multiplier or

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price update scheme is presented in this paper to improve convergence. In addition, this paper proposes an alternative algorithm based on lower bound and upper bound switching to have a faster convergence speed. The philosophy of the bound switching algorithm is similar as the philosophy of Bender's decomposition where lower bound and upper bound are computed. Details and refinement of the bound switching algorithm and the requirement of information exchange structure for demand response program are presented in this paper. Both algorithms are tested against multiple case studies: a radial network IEEE 399 system and two meshed networks (IEEE 30-bus system and IEEE 300-bus system).

The rest of the paper is organized as follows. Section 2 describes Lagrangian relaxation, Lagrangian dual problem and the concept of upper and lower bounds. Section 3 describes the decomposition of the utility and community optimization problems and the two demand response programs based on subgradient update and a bound switching algorithm. Section 4 presents numerical results and remarks. Section 5 concludes this paper.

2. Lagrangian relaxation based dual decomposition

2.1. Lagrangian relaxation

An optimization problem is generally defined as

$$\begin{aligned} f_0^* = f_0(x^*) = \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \le 0 \quad 1 \le i \le m \\ & h_j(\mathbf{x}) = 0 \quad 1 \le j \le p \end{aligned} \tag{1}$$

where $\mathbf{x}^*$ is the optimal solution of the decision variable vector $\mathbf{x}$. $f_0(\mathbf{x})$ is the objective function. $f_i(\mathbf{x})$ is an inequality constraint and $h_j(\mathbf{x})$ is an equality constraint.

Lagrangian relaxation technique relaxes the minimization problem by transferring constraints to objective function in the form of weighted sum as shown in (2).

$$L(\mathbf{x}; \lambda, \mu) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \mu_i f_i(\mathbf{x}) + \sum_{j=1}^{p} \lambda_i h_j(\mathbf{x}) \tag{2}$$

where λ_i and μ_i are Lagrangian multipliers or weights. μ_i should be greater or equal to zero.

Considering the Langrange dual function, $g(\lambda, \mu)$, as the greatest lower bound of $L(\mathbf{x}; \lambda, \mu)$, the Lagrange dual problem is defined as (3).

$$g^* = \max_{\{\lambda,\mu\}} g(\lambda, \mu) = \max_{\{\lambda,\mu\}} \{\inf_{\mathbf{x}} L(\mathbf{x}; \lambda, \mu)\} \tag{3}$$

According to the weak duality theorem, for any feasible solution (λ, μ) of the dual problem (3) and any feasible solution $\mathbf{x}$ of the original problem (1), the following relationship is true.

$$g(\lambda, \mu) \le g^* \le f_0(x^*) \le f_0(\mathbf{x}) \qquad \forall \mu \in \mathbb{R}_+^m, \lambda \in \mathbb{R}^p \tag{4}$$

Therefore, any feasible solution of the dual problem can result in a lower bound of the optimal value of the original problem (1).

2.2. Lower bound, upper bound, and gap

The definition of the upperbound (*UB*) and lowerbound (*LB*) must guarantee that $UB \ge f_0^*$ and $LB \le f_0^*$, respectively.

Indicated from the previous subsection, the cost corresponding to any feasible solution for the dual problem (3) is a **Lower Bound**.

On the other hand, since the optimal solution $\mathbf{x}^*$ for original problem (1) leads to the minimum cost, the resulting cost of any feasible solution $\mathbf{x}$ is an **Upper Bound**.

The difference between an upper bound and a lower bound which indicates the efficiency of the solution sought is called *Duality Gap* or *Gap*. (*Gap=UB–LB*).

3. System model and algorithms

Consider a power network consisting of a set *N* of buses and a set *B* of branches. The utility is responsible to operate the power grid, its generation units and transactions with transmission systems. Some community microgrids are connected to the network and behave as autonomous agents. The connected buses belong to a set *A*. The buses that belong to utility belong to a set $N - A$.

The communities share with the utility only limited information, which implies the following situations:

- Due to privacy issues, a community does not fully share information to the grid.
- Due to computing burden, the energy management center of a utility has no ability to collect every piece of information from customers. Instead, it is more feasible to have aggregated loads.
- The utility and the communities all behave as autonomous agents.

3.1. Lagrange relaxation and decomposition of optimal power flow problems

Optimal Power Flow (OPF), the well-known problem in power system operation, is defined in (5). It is obvious that an AC OPF takes care of not only active and reactive power balance constraints but also the other constraints such as voltage constraints, power line capacities, maximum and minimum limits of generators, etc.

$$\begin{aligned} \min \quad & \sum_{i \in N} C_i(P_{g_i}) \\ \text{subject to} \quad & \forall i \in N, \quad \forall j \in B \\ & P_{g_i} - P_{L_i} - P_i(V, \theta) = 0 \\ & Q_{g_i} - Q_{L_i} - Q_i(V, \theta) = 0 \\ & V_i^m \le V_i \le V_i^M \\ & P_{g_i}^m \le P_{g_i} \le P_{g_i}^M \\ & Q_{g_i}^m \le Q_{g_i} \le Q_{g_i}^M \\ & S_j(V, \theta) - S_j^M \le 0 \end{aligned} \tag{5}$$

where $C(\cdot)$ is the cost function, superscripts *M* and *m* denote upper and low bounds. Subscript *i* refers to the variables corresponding to bus *i*. P_g, Q_g, P_L and Q_L are the vectors of bus real and reactive power injection, and real and reactive loads. $P(V, \theta)$ and $Q(V, \theta)$ are the power injection expressions in terms of bus voltage magnitude and phase angles. $S(V, \theta)$ is the vector of line complex power flow.

Let us define two subscripts $(\cdot)_{imp_i}$ and $(\cdot)_{\exp_i}$ which are used in Fig. 1. The subscript $(\cdot)_{imp_i}$ denotes the utility's power import from a community connected to bus *i* while $(\cdot)_{\exp_i}$ denotes the same community's power export to utility. Hereafter in the paper, the community connected to bus *i* will be called community *i* for simplicity. In order to meet the power balance constraint, (6) must be fulfilled for all $i \in A$.

$$\begin{aligned} P_{imp_i} &= P_{\exp_i} \\ Q_{imp_i} &= Q_{\exp_i} \end{aligned} \tag{6}$$

In order to decompose the OPF problem between utility and communities, joint constraints (6) can be relaxed using Lagrange relaxation. In both utility's and communities' optimization problems, these constraints are not considered explicitly, but rather are