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Probabilistic load flow incorporating generator reactive power limit violations with spline based reconstruction method

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ABSTRACT

The increase in the penetration of intermittent generation and uncertainty in load patterns calls for the need to include these uncertainties in the conventional power flow programs. Consideration of the generator reactive power limit violation is an essential aspect in load flow studies for planning. In this paper, a probabilistic load flow method is proposed in which the violation of the reactive power limits of the generators are adequately represented. The seven point estimate method along with the spline based reconstruction technique is proposed for constructing the probability density function of the resulting multimodal distributions. The proposed load flow method has been tested on the IEEE-118 bus and IEEE-300 bus test systems for unimodal and multimodal load distributions, with and without correlation, and the accuracy of the results has been validated by comparing these results with those obtained by Monte Carlo simulation studies. Further, the effect of slack bus power limits has also been studied in this work.

1. Introduction

Presently the power system operates with many uncertainties which originate due to generation of power from renewable energy sources (RES) and variations in loads. These variations are going to be quite significant in future (if not already) because of significant and increasing penetration of RES in the grid and uncertainty in load patterns. Therefore, for successful operation of a power system, the effects of these possible variations need to be properly analysed, estimated and quantified. This can be achieved through load flow analysis of the grid in the presence of uncertain power generation from RES and uncertain load patterns. Towards this goal, various probabilistic load flow (PLF) methods have already been suggested in the literature [1–9].

Probabilistic Load Flow (PLF) was first proposed by Borkowska in 1976 [1] and thereafter was applied to different areas of planning and operation [2,3]. One of the approaches is to use the Monte Carlo simulation (MCS) method which involves running deterministic load flow thousands of times to get the probability distributions of the quantities of interest [4]. The results obtained from MCS are accurate as it uses exact non-linear load flow equations and are used as a benchmark for quantifying the effectiveness of other approximate PLF methods. But this method requires very high computational time. The use of hybrid latin hypercube sampling technique and Cholesky decomposition method has been proposed in [5] for better computational efficiency of Monte Carlo simulation.

Based on the concept that cummulants of a sum of random variables is equal to the sum of the individual cummulants of the random variables, PLF techniques using cummulants and Gram–Charlier expansion series [6] as well as cummulants and Cornish–Fisher expansion series [7] have been developed in the literature. Using cummulants, the convolution of random variables (RV) is reduced to the addition of the cummulants of RV. As is well known [8], the first four cummulants denote the mean, variance, skewness and kurtosis respectively. In the methods based on the cummulants, cross cummulants are zero for independent RV and also higher order (>4) cummulants can be neglected if the distribution of RV is near gaussian. An efficient method for estimating the cummulants of variables of interest is Point Estimate Method (PEM) as proposed in [9,10]. It is a good choice for the uncertainty analysis as it obtains the moments and cumulants of variables of interest by a weighted sum of the concentrations of input variables. The approximation of resultant probability density function (PDF) and cumulative distribution function (CDF) is then obtained through orthogonal series such as Gram–Charlier and Cornish–Fisher series. Between the Gram–Charlier and Cornish–Fisher series the former has a tendency to produce small negative frequencies particularly in the tails [11]. Further, as has been noted in [12], Gram–Charlier series suffers from serious convergence issues for non-Gaussian PDF.

PLF with RES has already been proposed in the literature in [7,12–17]. Out of these works, PDF and CDF of the quantities of interest have been calculated in [7,12,14–17], while in [13] only the mean and the standard deviations (of the quantities of interest)

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have been calculated using an extended point estimate method. Further, in all these works, the obtained results from the PLF have also been compared with the results obtained by MCS studies. In [14], Fourier Transform based convolution using DC power flow method has been used to find out the PDF and CDF of the variables of interest. On the other hand, in [7,12,14–16] initially the moments and cummulants of the quantities of interest have been calculated using a suitable technique (such as PEM [7,15] and weighted sum of cummulants of input variables [12,16]) and subsequently the PDF and CDF have been computed using an appropriate series expansion (such as Cornish–Fisher [7,15] and Gram–Charlier [12,16,17]). However, none of these above methods has considered the limits on the reactive power generated/absorbed by the generator, which is an important aspect of load flow studies.

To address the above issue, a method for considering the violations of generator reactive power limits in PLF is proposed in this paper. To compute the moments of variables of interest, PEM [9,10] has been used in this work for unimodal and multimodal load PDF with and without correlation between them. Subsequently, a spline based reconstruction technique has been used for computing the PDFs. To check the effectiveness of the proposed method, the computed PDFs have also been compared with PDFs obtained by using the Cornish–Fisher expansion series [7] and Monte-Carlo simulation studies. Also, the effect of slack bus power limit on the PDFs of variables of interest has been explored.

The paper is organized as follows. In Section 2, the basic procedure of PEM based PLF with the inclusion of generator reactive power limit is described using three point, five point and seven point estimate methods. In Section 3, the basic concept of spline based reconstruction technique is discussed. Lastly, in Sections 4 and 5, main results and conclusions of this work are presented, respectively.

2. Point estimate based PLF

The point estimate method can be used to calculate the statistical moments of a random quantity which, in turn, is a function of one or several random variables [9]. In this method, 'h' points on the PDF are first estimated and subsequently, from these estimated points the complete PDF is constructed. The general theory of 'h' point estimation method is given in [18]. However, in this present work 3, 5 and 7 point estimate methods have been used and the detailed procedure of PLF using these three methods is given below. For this purpose, it has been assumed that in a power system there are total 'n' number of random input variables. For instance, if there are 'L' load buses in a power system, each having both real and reactive power loads which are randomly fluctuating, then $n = 2L$. The objective of PLF is to calculate the PDFs of bus voltage magnitudes and angles from the PDFs of these n' variables.

Let the lth random variable $x_l (l = 1, 2, \ldots n)$ having PDF f_l be considered [10]. The PEM uses two, three or h' estimated points of x_l i.e. $x_{l,1}, x_{l,2}$ or $x_{l,h}$ as defined in eq. (1) to replace f_l by matching the first $h + 1$ moments of f_l.

$$x_{l,k} = \mu_l + \xi_{l,k}\sigma_l \quad \text{for } k = 1, 2 \ldots h \tag{1}$$

In Eq. (1), μ_l and σ_l are mean and standard deviation of x_l respectively and $\xi_{l,k}$ can be obtained as explained in the following sub-section for three, five and seven point estimate methods. The procedure for estimating the points of each variable x_l with their corresponding weights is described below.

2.1. Three, five and seven point estimate methods

1 Find the standard central moments as [9]:

$$\lambda_{l,i} = \frac{E[(x_l - \mu_l)^i]}{\sigma_l^i}, \quad i = 3, \ldots, 2m. \tag{2}$$

It is to be noted that, $m = 2$ for three point estimate method (3PEM), $m = 4$ for five point estimate method (5PEM) and $m = 6$ for seven point estimate method (7PEM).

2 Find the standard locations $\xi_{l,q}$, where $q = 1, \ldots m$. For 3PEM these locations are calculated by using eq. (3) while for 5PEM and 7PEM, these are calculated by obtaining the roots of the polynomial given in Eq. (4).

$$\xi_{l,k} = \frac{\lambda_{l,3}}{2} + (-1)^{3-k} \times \sqrt{\lambda_{l,4} - \frac{3}{4}\lambda_{l,3}^2}, \quad \xi_{l,3} = 0, \quad k = 1, 2. \tag{3}$$

$$p(\xi) = C_0 + \sum_{j=1}^{m} C_j \xi^j \tag{4}$$

In Eq. (4), $C_m = 1$ and the coefficients $C_0, C_1, \ldots, C_{m-1}$ are the solutions of the system of equations shown below:

$$\begin{bmatrix} 0 & 1 & \lambda_{l,3} & \cdots & \lambda_{l,m} \\ 1 & \lambda_{l,3} & \lambda_{l,4} & \cdots & \lambda_{l,m+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{l,m} & \lambda_{l,m+1} & \cdots & \cdots & \lambda_{l,2m-1} \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{m-1} \end{bmatrix} = - \begin{bmatrix} \lambda_{l,m+1} \\ \lambda_{l,m+2} \\ \vdots \\ \lambda_{l,2m} \end{bmatrix} \tag{5}$$

3 After the calculation of standard locations $\xi_{l,q}$, obtain $x_{l,q}$ from eq. (1), where, $q = 1, \ldots, m$. The weighting factors $w_{l,k}$ and $w_{l,3}$ for 3PEM are computed from Eq. (6) and the weighting factors $w_{l,q}$ and w_μ for 5PEM and 7PEM are determined by Eq. (5) and (8).

$$w_{l.k} = \frac{(-1)^{3-k}}{\xi_{l,k}(\xi_{l,1} - \xi_{l,2})}, \quad w_{l,3} = \frac{1}{n} - \frac{1}{\lambda_{l,4} - \lambda_{l,3}^2}, \quad k = 1, 2 \tag{6}$$

$$\begin{bmatrix} w_{l,1} \\ w_{l,2} \\ \vdots \\ w_{l,m-1} \\ w_{l,m} \end{bmatrix} = \begin{bmatrix} \xi_{l,1} & \xi_{l,2} & \cdots & \xi_{l,m} \\ \xi_{l,1}^2 & \xi_{l,2}^2 & \cdots & \xi_{l,m}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{l,1}^{m-1} & \xi_{l,2}^{m-1} & \cdots & \xi_{l,m}^{m-1} \\ \xi_{l,1}^{m} & \xi_{l,2}^{m} & \cdots & \xi_{l,m}^{m} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ \lambda_{l,3} \\ \vdots \\ \lambda_{l,m} \end{bmatrix} \tag{7}$$

$$w_\mu = 1 - \sum_{l=1}^{n}\sum_{k=1}^{m} w_{l,k} \tag{8}$$

2.2. PLF using PEM

Once the points and weights corresponding to 3PEM, 5PEM and 7PEM are estimated, the following procedure is adopted for PLF:

1 Form the input matrices $\boldsymbol{X_1}, \boldsymbol{X_2}, \ldots, \boldsymbol{X_k}$ as;

$$\boldsymbol{X_k} = \begin{bmatrix} x_{1,k} & \mu_{x2} & \cdots & \mu_{xn} \\ \mu_{x1} & x_{2,k} & \cdots & \mu_{xn} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{x1} & \mu_{x2} & \cdots & x_{n,k} \end{bmatrix} \tag{9}$$