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A direct method based on tensor calculation to determine maximum loadability power flow solutions

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ABSTRACT

In this paper, the determination of critical loadability points of the power flow equations is formulated as an optimisation problem. A quadratic parameterisation scheme is used to model the load change, which is equivalent to implicitly including a non-negativity constraint in the load variation. The power flow equations are expressed in rectangular coordinates. This is suitable to exploit the second order information (tensor term) of the equations representing the optimality conditions. The use of this term improves the convergence of the iterative process and facilitates the manipulation of the reactive power generation limits. Simulation results obtained with a number of power systems, including real networks, are used to illustrate the main features of the proposed methodology.

1. Introduction

Several aspects of voltage instability have been explained by the Bifurcation Theory of nonlinear systems. In particular, two types of local bifurcations have been frequently associated with the steady state aspects of the voltage collapse [1,2]. The first is the *saddle-node* bifurcation, in which the Jacobian matrix of the differential-algebraic equations representing the power system is singular [3,4]. The second, named *limit-induced* bifurcation, is related to changes in the stability condition, as a consequence of a particular variable reaching its limit. In the present work, the power flow solution corresponding to the maximum (here also referred to as *critical*) loadability is characterised as a saddle-node bifurcation point (also named limit point, fold point and turning point). Although very interesting from the practical point of view of power system stability, other types of bifurcation are not focused on here.

A number of methods for determining the critical loadability point can be found in the literature. Continuation methods have been widely used to determine a sequence of solutions for the parameterised power flow equations from a base load to the point of maximum loadability [5–7]. A predictor-corrector strategy is applied to calculate the set of equilibrium points that compose the PV-curve of each bus. Additionally, the predictor tangent vector is obtained as a by-product of the computational process, with reduced computational effort through the use of the techniques presented in [8]. Direct methods determine the saddle-node bifurcation point in one step, providing simultaneously the right or left eigenvectors of the singular power flow Jacobian matrix. Some of the relevant approaches found in the literature are mentioned as follows. In [9], an optimisation problem is solved by Newton's method, in which (without loss of generality) the active power level is constant, such that only the QV subproblem is solved. In [7,10], the set of equations representing the Transversality Conditions are extended to integrate HVDC links, area interchange power control and other power systems particularities. The resulting set of nonlinear equations is solved to provide the so-called *Point of Collapse*. Refs. [11,12] propose direct methods to compute the saddle-node bifurcation point closest to the base case in the load parameter space. These methods combine the computation of both the load power margin and the vector normal to the limit surface of the feasible region of the power flow equations. They can provide the saddle-node bifurcation point in a fixed direction of load increase as well as the bifurcation point locally closest to the current operating load level. Ref. [13] proposes the determination of the loadability margin by solving the Moore–Spence augmented system. Its major contribution is the application of a matrix decomposition strategy to speed up the computational process. Ref. [14] combines the Continuation method with optimisation techniques to detect the limit of solvability of the power flow equations. A least squares technique is used to calculate the Lagrange multipliers, which are used as a figure of merit to find the power flow solvability limit. More recently, Yang et al. [15] proposes the use of block elimination techniques to solve the Moore–Spence augmented system. Similarly

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to [13], the contribution of this approach is related to the implementation aspects. The similarities and differences between these methodologies concern five main points: (1) the analytical formulation, (2) the type of load parameterisation, (3) the solution method, (4) the initial estimates, and (5) the by-products of the computation. First, the sets of equations solved to calculate fold points are basically those which represent alternatively the Transversality Conditions [7,12] the Moore–Spence determining system [13,15], or the optimality conditions of the maximum loadability optimisation problem [9]. In case of the optimisation model, adequate assumptions must be taken into account in order to obtain fold points similar to those produced by Continuation methods. Except for this aspect there are no considerable differences between these models. Second, with respect to the parameterisation, all of these methodologies have used the linear power load parameterisation to model the load increase. Unless a non-negativity constraint on the load parameter is added to the formulation, there is a risk of determining undesirable critical loadability solutions with load decrease (negative load parameter), if the initial estimate is not well chosen. Referring to the solution method, due to the particular structure and features of the maximum loadability problem, all of these approaches apply Newton's method to solve the set of nonlinear equations that provide the fold point. This method is well defined and quadratically (although only locally) convergent. It is efficient to deal with equality and inequality constraints [9], but requires a good initial estimate. For this purpose, strategies like iterations of the Inverse Power Method have been used in [7,10], information about the largest eigenvalue of the Jacobian matrix available in the current operating point is employed in [11,12] and the Continuation method is applied in [13–15] to provide the initial guesses for power flow variables and eigenvectors. From this point of view, the exception is the approach presented in [9], which claims no need for the solution of any separate system to obtain improved initial estimates. The numerical results obtained through the application of these direct methods include the fold point and the sensitivity relationships represented by the eigenvectors and Lagrange multipliers, which are used to define critical buses and areas.

The maximum loadability problem can be also formulated as a general Optimal Power Flow problem, as in [16,17], for example. The analytical models used in these approaches include other types of operational constraints (voltage magnitude, power flows, etc.) and optimisation variables (generated active and reactive powers, transformer taps, etc.). This brings additional difficulties (in terms of size and complexity of the problem) to obtain critical solutions and requires different numerical methods to solve the optimisation problem (as Interior Point Methods and/or Trust Region based-methods, for example). The maximum loadability solutions produced through the application of these methodologies is completely different from those focused on the present work. For this reason this type of approach is out of the scope of this paper.

The present paper proposes a methodology for the direct determination of the turning points of the power flow equations. This is formulated as a constrained static optimisation problem, which is solved by an extended version of Newton's method. For this purpose, the power flow equations are expressed in rectangular coordinates. The main contributions reported here are: (a) the use of an alternative load parameterisation scheme to model the load variation, so as to ensure power flow solutions with load level greater than that of the base case; (b) the inclusion of the second-order information (tensor component) in the search direction, to improve the robustness of the iterative process and facilitate the treatment of the reactive power generation limits. The proposed approach differs from those previously mentioned with respect to: (1) the type of load parameterisation, (2) the strategy to solve the optimisation problem, which is an extension of Newton's method to exploit the second-order information. Similarly to [9], there is no need of any special strategy to choose initial guesses for the Lagrange multipliers, although we also show two alternative procedures to compute the initial estimates of the Lagrange multipliers. Numerical results obtained for power systems with sizes ranging from 24 to 1916 buses are used to illustrate the proposed methodology.

This paper is organised as follows. Section 2 describes the theoretical basis of the maximum loadability problem, including the solution through static optimisation algorithms. Section 3 presents the analytical model proposed herein, with emphasis on the use of the information conveyed by the tensor term. Section 4 presents the numerical results obtained from the application of the methodology proposed here and Section 5 summarises the main conclusions.

2. Preliminary concepts

The determination of the power flow solution at the maximum loadability level can be formulated as an optimisation problem expressed in terms of power system variables as:

$$\begin{array}{ll} \text{Maximize} & \rho \\ \text{subject to} & (P_{g_i}^0 + \rho^2 \Delta P_{g_i}) - (P_{d_i}^0 + \rho^2 \Delta P_{d_i}) - P_i(e,f) = \ \ 0 \quad \text{(PV, PQ buses)} \\ & Q_{g_i}^0 - (Q_{d_i}^0 + \rho^2 \Delta Q_{d_i}) - Q_i(e,f) = \ \ 0 \quad \text{(PQ buses)} \\ & V_i^{ref^2} = e_i^2 + f_i^2 \quad \text{(PV buses)} \\ & Q_{g_i} = (Q_{d_i}^0 + \rho^2 \Delta Q_{d_i}) + Q_i(e,f) \le Q_{g_i}^{lim} \quad \text{(PV buses)} \end{array} \tag{1}$$

where $P_{g_i}^0$ and $Q_{g_i}^0$ are the active and reactive power generation of bus i in the base case and ΔP_{g_i} represents the active power generation change rate, which indicates how the active power generation follows the increase of the loadability level (distribution factors based on economic and/or security criteria can be used for this purpose); $P_{d_i}^0$ and $Q_{d_i}^0$ refer to the active and reactive power demand of the ith bus in the base case, ΔP_{d_i} and ΔQ_{d_i} represent the prespecified direction of increase of the active and reactive power load of bus i, V_i^{ref} is the reference voltage magnitude of the ith PV bus, and $P_i(e,f)$ and $Q_i(e,f)$ are the nodal active and reactive power injections expressed as functions of the real and imaginary components of complex voltages. The inequality constraints refer to limits of generated reactive power of the PV buses. The optimisation variables are the components of the complex voltage (e_i, f_i) and the load parameter ρ.

If the conventional linear parameterisation is used to model the load variation and a non-negativity constraint is not imposed on ρ, the optimisation problem of Eq. (1) can provide undesirable critical solutions (with negative loading factor, for example), which are useless from the practical point of view. This is usually due to a poor initial estimate of the optimisation variables, in particular the Lagrange multipliers. In order to overcome this difficulty, the load change is quadratically parameterised, as shown in Eq. (1). This type of parameterisation is equivalent to imposing a non-negativity constraint in the load variation.

The constraints of the optimisation problem of Eq. (1) are expressed in compact form as:

$$\begin{array}{rcl} \mathbf{g}(\mathbf{x}, \rho) & = & \mathbf{y}_0 + \rho^2 \mathbf{r} - \mathbf{g}_0(\mathbf{x}) = \mathbf{0} \\ \mathbf{h}_{q_g}(\mathbf{x}, \rho) & = & \mathbf{y}_{q_0} + \rho^2 \mathbf{r}_{q_g} - \mathbf{h}_0(\mathbf{x}) \le \ \ \mathbf{h}_{q_g}^{lim} \end{array} \tag{2}$$

where the components of vectors $(\mathbf{y}_0 + \rho^2 \mathbf{r})$ and $(\mathbf{y}_{q_0} + \rho^2 \mathbf{r}_{q_g})$ are related to the bus power injections, the squared voltage magnitude of the PV buses in the base case ($\mathbf{y}_0$ and $\mathbf{y}_{q_0}$) and the load increase direction ($\mathbf{r}$ and $\mathbf{r}_{q_g}$). Since the PV bus voltage magnitude does not depend on the load variation (see the third equality constraint in Eq. (1)), the components of the vector $\mathbf{r}$ corresponding to the squared voltage magnitudes are zero. Vectors $\mathbf{g}_0(\mathbf{x})$ and $\mathbf{h}_0(\mathbf{x})$