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Accurate numerical method to evaluate the capacitances of multi-conductor power cables

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ABSTRACT

This paper presents an accurate method to evaluate the capacitances of multi-conductor power cables. The method developed can also be applied to cables presenting multilayer dielectrics. Strips of charge are used to represent the electric charge distribution on conductors' surface, as well as to characterize the polarization charge distribution on interfaces separating two different dielectrics. The advantage of using strips of charge is to preserve charge continuity along conductor surfaces and dielectric interfaces. This methodology is accurate since the proximity effects among conductors are taken into account. Validation of the method is performed comparing experimental, other software and analytical results with those obtained by using the proposed numerical method. A cylindrical non-concentric capacitor is used for the analytical validation and a three-phase power cable is used on experimental tests.

1. Introduction

This paper presents an accurate method to evaluate the capacitances of multi-conductor power cables. The method can be extended to cables presenting multi-layered dielectrics.

Longitudinal impedances and transversal admittances of power transmission lines, either overhead or underground, in the frequency domain, have to be determined more and more accurately in order to attain efficient and rigorous simulations [1,2]. Analytical solutions can be applied if simple physical systems are present [3–7]. For a general case, where the geometry has a complex configuration or the material electromagnetic properties have heterogeneities, anisotropies or even non-linearity, it is not suitable to use analytical solutions. In such cases, numerical methods must be applied. The most important numerical methods [8] have been the Finite Difference Method, the Finite Element Method [9], the Boundary Element Method [10,11], the Method of Moments (MoM) [12–15], the Charge Simulation method (CSM) [16,17] and the Optimized Charge Simulation Method (OCSM) [18,19].

In this paper, a 2D electrostatic field problem is considered. The MoM is developed where the integral operator has, in this case, the form of a surface integral which is inherent to the presence of surface electric charge distributions. The MoM leads to identical formulation as the CSM. The version proposed in this paper takes the equivalent charges on the conductor surfaces with the form of strip charges instead of line charges located somewhere inside the conductors as done in a typical CSM. In [13], as well a MoM approach is used to deal with finite length coaxial cables in the proximity of a conductor plane but where proximity effect is neglected assuming uniform charge densities along each element interface. In this paper, a 2D approach is used to deal with infinitely long cables where proximity effect is considered. For this aim, strips of charges are used to represent the electric charge distribution on conductors' surface as well as to characterize the polarization equivalent charge distribution on the separation surface between two different dielectric regions.

In this way, the method developed in the paper is appropriate to deal with linear and isotropic dielectric media, the permittivity being considered as a real parameter, but where heterogeneities, resulting from the presence of interfaces separating homogeneous regions, may be taken into account.

The point-matching technique is applied in the MoM [12], meaning that Dirac's functions are used for the weighting functions [12] and, on the other hand, linear approximations or triangle functions are used for the sub-sectional basis functions instead of pulse functions adopted in the typical applications of the MoM [12]. This means that strip charges are approximated to a linear variation along the strip width. This approximation has the form of a linear combination of the charge surface density on the edges of the strips taken as discretization nodes in the 2D description of the problem. The charge surface density in each node is evaluated imposing boundary condition in the Dirichlet form (imposing the electric potential) on the conductor surface. As a consequence of the equivalent charge location, singularities must appear in the

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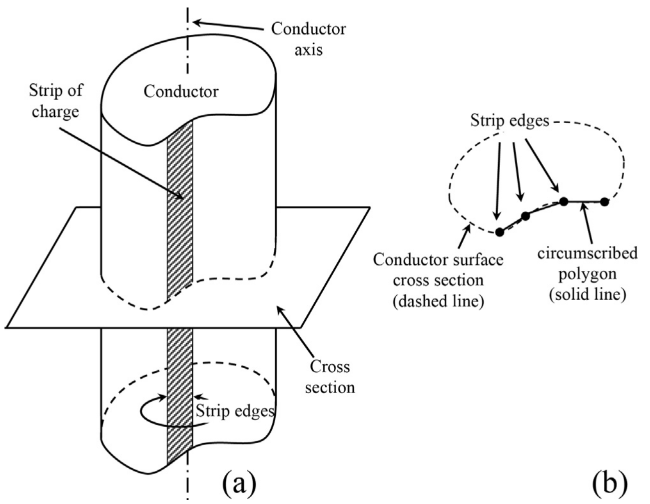

Fig. 1. Representation of the strip of charge concept: (a) representation of one strip of charge into which the conductor surface is subdivided. The conductor surface cross section, obtained using a plane orthogonal to the conductor axis is represented by a dashed line; (b) the conductor surface cross section (dashed line) is approximated to a circumscribed polygon (solid line). One side of the polygon is represented in (a) as the cross section of the strip of charge.

integrand functions. In the paper, those singularities are treated analytically.

The method is validated by comparing numerical results with other software, analytical and experimental ones [20]. A cylindrical non-concentric capacitor is used for the analytical validation. For experimental tests, a three-phase power cable is used.

Section 2 deals with the description of the method, first for conductors inside a homogeneous dielectric and then for cases with dielectrics separated by interfaces. Numerical results are presented in Section 3. Finally, in Section 4, conclusions are presented.

2. Electric field solution due to a system of parallel conductors

2.1. Conductors in homogeneous dielectric medium

Under the electrostatic regime, the electric field inside a charged conductor is zero and consequently the conductor's electric charge is distributed along its boundary surface, with a density ω ([Cm^{-2}]). A system of parallel conductors is considered inserted in a homogeneous dielectric medium. The methodology developed for the 2D electric field problem approaches the conductor surface cross section by a circumscribed polygon, Fig. 1. Each side of the polygon line in the cross section corresponds in fact to a strip of charge extruded along the conductor surface parallel to the conductor axis. The conductor surface charge is, therefore, characterized by a mesh of strips of charge, where at their edges the charge density is defined by the charge on conductor's surface located at the same position as the strip edges. Strip edges (Fig. 1) correspond to the connection points between adjacent strips where the continuity of the charge density is observed. Strip edges correspond, in this way, to the discretization nodes in the 2D description of the problem.

The MoM is applied where the integral operator has, in this case, the form of a line or path integral which is inherent to the presence of 2D surface electric charge distributions. The point-matching technique is adopted [12], meaning that Dirac's functions are used for the weighting functions and, on the other hand, linear approximations are used for the sub-sectional basis functions instead of step functions [12]. This means that strip charges are approximated to a linear variation along the strip width.

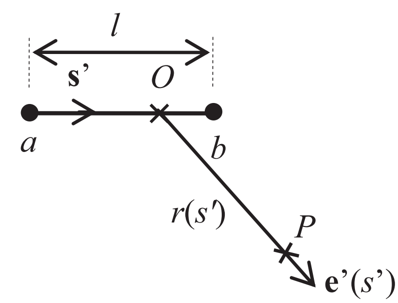

Fig. 2. Cross section of a strip of charge defined by the path **s'** between the edge nodes a and b. The point P is represented by the distance $r(s')$ to the point O of the path **s'**. Location of point O is defined by the magnitude s' which represents the separation distance between point O and the origin of the path **s'** – point a.

Consider a strip whose cross section is represented in Fig. 2, where a linear charge distribution is assumed. The strip is described by a path, in this case by the path **s'**. If the charge density is equal to ω_a and ω_b at the edge points a and b, respectively, then the charge density along the strip is given by:

$$\omega(s') = f_a(s')\omega_a + f_b(s')\omega_b$$
$$f_a(s') = -\frac{1}{l}s' + 1; \quad f_b(s') = \frac{1}{l}s's \tag{1}$$

where s' represents the magnitude of path **s'** given by the distance between a point on the strip and the respective path's origin, point a, and l represents the width of the strip – the distance between points b and a.

The potential at the free space point P due to this strip of charge is given by:

$$V(P) = \int_{s'} \frac{\omega(s')}{2\pi\varepsilon_0} \ln\left(\frac{1}{r(s')}\right) ds' + V' \tag{2}$$

where, ε_0 is the vacuum permittivity, V_0' is an integration constant and, from Fig. 2, $r(s')$ is the distance between the observation point P and the integration point on the strip of charge, O. Location of point O is defined by the magnitude s' of the strip path **s'**. This integral has an analytical solution [21] (see Appendix), and the result, for each point P where the potential is evaluated, depends not only on the charge density at the strip ends (ω_a and ω_b) but also on the position of P relative to the strip of charge. Note that the solution in (2) corresponds to the usage of the 2D Green's function for the free space.

The electric field at P due to this strip of charge is given by:

$$E(P) = \int_{s'} \frac{\omega(s')}{2\pi\varepsilon_0} \frac{1}{r(s')} e(s')\, ds' \tag{3}$$

where $e(s')$ is the unit vector along the direction defined by the point P and the point O of the strip of charge (Fig. 2). This integral can also be evaluated analytically [21] (see Appendix).

The potential due to N charge strips is obtained by superposition, adding the contribution of each strip individually as a consequence of the linearity of the problem.

In this way, the global solution for the electric potential may be built using the following global decomposition for the surface charge density approximation

$$\omega(s) = \sum_{k=1}^{N} f_k(s)\,\omega_k \tag{4}$$

where s represents the magnitude of the global path **s** defining the conductor surfaces approximated to polygonal lines

$$s = s_1 \cup s_2 \cup \cdots \cup s_N \tag{5}$$