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Solving the hydrothermal scheduling problem considering network constraints

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ABSTRACT

This paper addresses the Short-Term Hydrothermal Scheduling – STHS problem of power systems. In this problem, besides the constraints associated to hydro and thermal plants, network constraints are included due to the significant number of hydro plants located far from the load. In the STHS problem, hydro and thermal plants must be coordinated in order to supply the demand at a minimum cost and comply with a set of constraints, over a short-term horizon. Aiming to solve this problem considering the presence of nonlinearities, large number of decision variables and constraints coupled in time periods, we propose a strategy based on Lagrangian Relaxation – LR and Augmented Lagrangian – AL techniques. By using LR scheme, based on a variable splitting technique, the resulting separable dual problem is solved by a Bundle method. A primal feasible solution is obtained using the AL technique, based on the Auxiliary Problem Principle – APP approach, starting from the primal and dual solutions supplied by the LR phase. A computational model that makes use of the proposed strategy is tested on a hydrothermal configuration, whose data were extracted from the Brazilian Hydrothermal power system.

1. Introduction

The operation planning of hydrothermal systems is a very complex task. A possible approach involves the decomposition of the global task into long, medium and short-term scheduling problems [1,2]. In general, uncertainties are modeled in detail in the long and medium-term problems, and the system and generation constraints are precisely detailed in the short-term problem. In this paper, the Short-Term Hydrothermal Scheduling – STHS is addressed. This problem has been intensely researched and numerous approaches have been proposed [3–12]. Due to the significant number of coupling constraints and decision variables, decomposition techniques come out as alternatives; particularly, the Lagrangian Relaxation – LR, which is one of the most efficient strategies [8–13]. Since we are concerned with systems with significant participation of the hydro generation, these plants must be modeled in details, so that we can obtain a practical solution, avoiding discretionary adjustment in the real time operation. In addition, frequent start-ups and shutdowns of hydro units are usually not allowed because of the resulting mechanical stress. Thus, aiming to avoid this undesirable operation, in this work, minimum up and downtime constraints are modeled for hydro units. The consideration of the electrical network is important to obtain solutions that are feasible in practical terms. In systems with long transmission lines, congestions come out frequently, which may change the merit order of the economic dispatch. In order to consider these constraints a DC model for the transmission network is included.

As we aforementioned, several researches in this area have been developed. However, these studies always have some simplifications in the generation or in the transmission modeling. In [12], the thermal generation and the network constraints are modeled in detail, but the hydro production function is simplified and the forbidden operating zones are not modeled. In [14], only the hydro generation is modeled, so the thermal and the network constraints are not modeled. This way, in this paper, we address the STHS problem which includes in a single computational model, the hydrothermal unit commitment and the transmission constraints.

Regarding the solution strategy, by using LR scheme the original STHS problem is split into a sequence of smaller and easy-to-solve subproblems, coordinated by a dual master problem. The advantage of our approach is to obtain five separate subproblems: thermal, hydro, hydrothermal, hydro unit commitment and medium-term coordination. Each subproblem is solved with a specific mathematical programming technique, as it will be described ahead. As LR method usually fails to find a feasible solution, a primal recovery phase is also used and we use an Augmented Lagrangian – AL method. One advantage of the AL over the LR is that the former may obtain a feasible solution, in cases where primal solutions present a duality gap. Nevertheless, one of the main drawbacks of the AL is that a quadratic term is introduced into the objective function

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making the problem not separable, which requires some heuristics to overcome this difficulty. This paper makes use of the Auxiliary Problem Principle – APP [15] approach. The main contributions of this paper are the high accuracy in the modeling generation and transmission systems. The proposed strategy in this paper is suitably designed to deal with: (a) forebay level varying according to the volume in the reservoir; (b) minimum up and downtime constraints for the hydro units; and (c) coordination with the medium problem by means of a Future Cost Function. Additionally, as an important improvement, a pseudo-primal point [16] is also used as a warm starting by the AL to find a feasible solution.

This paper is organized as follows: Section 2 presents the STHS notation; Section 3 presents the STHS formulation; Section 4 details the proposed solution strategy; Section 5 reports the hydrothermal configuration and the computational results; and, finally, Section 6 presents the main conclusions of this paper.

2. STHS notation

The nomenclature used to model the STHS optimization problem is defined below:

T	total stages, [h];
t	stage index, so that $t = 1,T$;
R	number of reservoirs;
r	index of reservoirs, so that $r = 1,R$;
v_{rt}	volume of reservoir r at the beginning of stage t [h m^3];
c_1	conversion factor of [m^3/s] into [h m^3];
s_{rt}	spillage of reservoir r during stage t [m^3/s];
$\Re^+(r)$	set of reservoirs immediately upstream of reservoir r;
τ_{mr}	water traveling time between reservoirs m and r [h];
y_{rt}	incremental inflow of reservoir r during stage t [m^3/s];
Q_{rt}	plant turbined outflow in reservoir r during stage t [m^3/s];
d_{rt}	total outflow in reservoir r during stage t [m^3/s];
J_{rt}	total of hydro units of reservoir r, during stage t;
j	index of hydro units, $j = 1,J_{rt}$;
q_{jrt}	turbined outflow of unit j of reservoir r during stage t [m^3/s];
$ph_{jrt}(.)$	power output of unit j, reservoir r and stage t [MW];
Φ_{jr}	total of forbidden operative zones of unit j of reservoir r;
k	index of forbidden operative zones, $k = 1,\Phi_{jr}$;
$ph_{jkrt}{}^{\min,\max}$	minimum (maximum) power of unit j, reservoir r and stage t, operating zone k [MW];
z_{jkrt}	binary variable that indicates if unit j of reservoir r is operating ($z_{jkrt} = 1$) or not ($z_{jkrt} = 0$) in zone k during stage t;
z_{jro}, w_{jro}	initial condition for hydro unit j of reservoir r;
$\tilde{z}_{jr0}$, $\tilde{w}_{jr0}$	initial values for z_{jro}, w_{jro}, respectively;
$t_{jr}{}^{up}$, $t_{jr}{}^{down}$	minimum uptime ($t_{jr}{}^{up}$) and downtime ($t_{jr}{}^{down}$) for hydro unit j of reservoir r [h];
w_{jrt}	variable that indicates the number of stages that the unit j of reservoir r has been on or off until the stage t;
$v_r{}^{\min,\max}$	minimum (maximum) volume of reservoir r [h m^3];
α	expected future cost corresponding to the stage $T+1$ on [\$];
p	index related to the piecewise linear Future Cost Function;
π_{rp}	constant factor associated with the reservoir r and the segment p of piecewise linear Future Cost Function [\$/h m^3];
$\underline{\alpha}_p$	independent term associated with the p piecewise linear Future Cost Function [\$];
n_t	number of thermal plants;
i	index of thermal plants, $i = 1,n_t$;
pt_{it}	power output of thermal unit i, during stage t [MW];
$c_{1i}, \ldots, c_{3i}$	quadratic function constants of thermal unit i generation cost;
pt_{i0}	initial power output of thermal unit i [MW];
u_{io}, x_{io}	initial condition of thermal unit i;
pt_0, $\tilde{u}_{i0}$, $\tilde{x}_{i0}$	initial values for pt_o, u_{io}, x_{io}, respectively;
Δ_i	ramp rate maximum of thermal unit i [MW];
$pt_{it}{}^{\min,\max}$	minimum (maximum) power of unit i and stage t [MW];
u_{it}	binary variable that indicates if thermal unit i is operating ($u_{it} = 1$) or not ($u_{it} = 0$) during stage t;
$t_i{}^{up}$, $t_i{}^{down}$	minimum uptime ($t_i{}^{up}$) and downtime ($t_i{}^{down}$) for thermal unit i [h];
x_{it}	variable that indicates the number of stages that the thermal unit i has been on or off until the stage t;
n_b	total of buses in the system;
b	index of bus;
I_b, R_b	sets with all thermal/hydro plants of bus b;
Ω_b	number of buses interconnected with bus b;
$\theta_{bm,t}$	phase angle difference between the buses b (θ_{bt}) and m (θ_{mt}) during stage t [rad];
x_{bm}	line reactance between buses b and m [pu];
$\overline{f_{bm}}$	maximum power interchange between the buses b and m [MW];
L_{bt}	hourly power demand of bus b, during stage t [MW];
$PH_{rt}{}^{\max}$	maximum hydro generation of plant r and stage t [MW];
RH_t	hydro reserve during stage t [MW].

3. STHS formulation

The STHS optimization problem related to this work is described as follows:

$$\min f = \sum_{t=1}^{T}\sum_{i=1}^{nt}[(c_{1i}pt_{it}^2 + c_{2i}pt_{it} + c_{3i})u_{it} + st_{it}(x_{i,t-1})u_{it}(1-u_{i,t-1})] + \alpha \tag{1}$$

$$\text{s.t.}: \quad v_{r,t+1} - v_{rt} + c_1\left(d_{rt} - \sum_{m\in\Re_+^{(r)}} d_{m,t-\tau_{mr}} - y_{rt}\right) = 0 \tag{2}$$

$$Q_{rt} + s_{rt} - d_{rt} = 0, \quad s_{rt} \le s_r^{\max}, \quad 0 \le Q_{rt} \le Q_r^{\max}, \quad d_r^{\min} \le d_{rt} \le d_r^{\max}, \quad v_r^{\min} \le v_{rt} \le v_r^{\max} \tag{3}$$

$$Q_{rt} - \sum_{j=1}^{J_{rt}} q_{jrt} = 0, \quad 0 \le q_{jrt} \le q_{jrt}^{\max} \tag{4}$$

$$\sum_{k=1}^{\Phi_{jr}} ph_{jkrt}^{\min} z_{jkrt} \le ph_{jrt}(q_{jrt}^7, d_{rt}^{12}, v_{rt}^{12}) \le \sum_{k=1}^{\Phi_{jr}} ph_{\mathrm{jkrt}}^{\max} z_{jkrt} \tag{5}$$

$$\sum_{k=1}^{\Phi_{jr}} z_{jkrt} \le 1, \quad z_{jrt} \in \{0,1\}, \quad z_{jr0} = \tilde{z}_{jr0}, \quad w_{jr0} = \tilde{w}_{jr0}, \tag{6}$$

$$z_{jrt} = \begin{cases} 1 & \text{if} \quad 1 \le w_{jrt} \le t_{jr}^{up} \\ 0 & \text{if} \quad -1 \ge w_{jrt} \ge -t_{jr}^{down} \\ 0 \ \text{or} \ 1 & \text{otherwise} \end{cases}, \qquad w_{jrt} = \begin{cases} \max(w_{jr,t-1},0)+1 & \text{if} \quad z_{jrt}=1 \\ \min(w_{jr,t-1},0)-1 & \text{if} \quad z_{jrt}=0 \end{cases} \tag{7}$$

$$\alpha + \sum_{r=1}^{R} \pi_{rp} v_{r,T+1} \ge \underline{\alpha}_p \tag{8}$$

$$pt_{i0} = p\tilde{t}_0, \quad u_{i0} = \tilde{u}_{i0}, \quad x_{i0} = \tilde{x}_{i0}, \quad |pt_{it} - pt_{i,t-1}| \le \Delta_i, \quad pt_{it}^{\min}u_{it} \le pt_{it} \le pt_{it}^{\max}u_{it} \tag{9}$$

$$u_{it} = \begin{cases} 1 & \text{if} \quad 1 \le x_{it} \le t_i^{up} \\ 0 & \text{if} \quad -1 \ge x_{it} \ge -t_i^{down} \\ 0 \ \text{or} \ 1 & \text{otherwise} \end{cases}, \qquad x_{it} = \begin{cases} \max(x_{i,t-1},0)+1 & \text{if} \quad u_{it}=1 \\ \min(x_{i,t-1},0)-1 & \text{if} \quad u_{it}=0 \end{cases} \tag{10}$$

$$\sum_{i\in I_b} pt_{it} + \sum_{r\in R_b}\sum_{j=1}^{J_{rt}} ph_{jrt}(q_{jrt}^7, d_{rt}^{12}, v_{rt}^{12}) - \sum_{m\in\Omega_b} x_{bm}^{-1}\theta_{bm,t} = L_{bt} \tag{11}$$