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Loss minimization by the predictor–corrector modified barrier approach

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ABSTRACT

This paper presents a new approach, predictor–corrector modified barrier approach (PCMBA), to minimize the active losses in power system planning studies. In the PCMBA, the inequality constraints are transformed into equalities by introducing positive auxiliary variables, which are perturbed by the barrier parameter, and treated by the modified barrier method. The first-order necessary conditions of the Lagrangian function are solved by predictor–corrector Newton's method. The perturbation of the auxiliary variables results in an expansion of the feasible set of the original problem, reaching the limits of the inequality constraints. The feasibility of the proposed approach is demonstrated using various IEEE test systems and a realistic power system of 2256-bus corresponding to the Brazilian South-Southeastern interconnected system. The results show that the utilization of the predictor–corrector method with the pure modified barrier approach accelerates the convergence of the problem in terms of the number of iterations and computational time.

1. Introduction

The optimal power flow (OPF) research has attracted a lot of attention of the utilities, due to the active power losses and the limited capacity of the transmission system to accommodate additional loads and to maintain a voltage profile and power flow adaptable to the different operational scenarios. The OPF can be used as an effective tool to reach all these goals, bringing economy and also a better performance of power system.

Since the OPF was formulated in [1], various optimization techniques, such as nonlinear and quadratic programming, Newton-based, linear programming and interior point method (IPM) have been proposed to solve the nonlinear OPF problem shown in [2,3]. Among earlier techniques applied to solve the OPF problem, the IPM is considered efficient, mainly due to its ease of handling inequality constraints and good performance to solve the problem. On the other hand, the IPM presents two drawbacks: the choice of the initial barrier parameter and the factorization of the Hessian matrix per iteration. In order to overcome these drawbacks, the predictor–corrector algorithm proposed by Mehrotra [4] has been associated with the IPM. Mehrotra's algorithm computes more successful search directions, usually leading to fewer iterations and less solution time, by performing one predictor and one corrector step per iteration. The first implementation of the IPM applied to the OPF problem was proposed in [5]. In the same year, the Primal-dual Interior Point algorithm with the predictor–corrector method [6] was used to accelerate the convergence of the problem. In recent years, most researches involving the OPF problem have been based on the variants of the IPM mainly with the predictor–corrector [7–19], and a few researches have been based on different approaches [20–26]. In [25] the modified barrier-augmented Lagrangian method [27], a variant of the modified barrier method [28], was used for the optimum selections of the transformers' tap positions and the voltage points of the generators, and in [26], the modified barrier (MB) method was used to establish the pricing mechanism for finding the equilibrium in an auction market.

The MB method is based on the modified barrier function (MBF) introduced in [28]. The MBFs have several characteristics, such as these functions and their derivatives are defined in the solution and do not grow infinitely. The barrier parameter does not need to be driven to zero and the Hessian matrix of the MBF does not become ill-conditioned when the current approximation approaches the solution. Another interesting characteristic of the MBF is the explicit representation of its Lagrange multiplier, which helps the convergence of the method.

Motivated by the efficiency of the MB and predictor–corrector methods, we propose the PCMBA to minimize the active power losses in transmission. Tests using the 30, 118, 300-bus and a 2256-bus system corresponding to the Brazilian South-Southeastern

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interconnected system electrical were carried out to show the efficiency of the proposed approach.

The paper is organized as follows: initially, the modified barrier method and the predictor–corrector modified approach are discussed. Then, the test results of the comparative study in four systems are reported. Finally, some concluding comments are presented.

2. Modified barrier method

In 1992, Polyak [28] developed the theory of the MB methods. These methods are combinations of the best properties of the Classic Lagrangian and the Logarithm Barrier functions (LBF), but they are free from their most essential drawbacks. The MB method has a finite convergence property as opposed to an asymptotic one for methods based on the LBF. It also enables constraints to become precisely equal to zero, including the boundary of the original optimization problem.

The development of the MBF is presented, as follows, considering the next nonlinear programming problem.

$$\begin{aligned}&\text{Minimize } f(\boldsymbol{x})\\&\text{subject to } \boldsymbol{h}(\boldsymbol{x}) \geq 0\end{aligned} \quad (1)$$

where $\boldsymbol{x} \in R^n$, $\boldsymbol{h}(\boldsymbol{x}) \in R^p$, and, f and $\boldsymbol{h}$ are continuously differentiable functions in R^n with values in R and R^p.

In order to obtain the definition of the MBF, the problem (1) undergoes various modifications. First, the barrier parameter, μ, in the inequality constraint is added.

$$\begin{aligned}&\text{Minimize } f(\boldsymbol{x})\\&\text{subject to } \mu + \boldsymbol{h}(\boldsymbol{x}) \geq \mu\end{aligned} \quad (2)$$

Second, the inequality constraint is divided and increased by the barrier parameter.

$$\begin{aligned}&\text{Minimize } f(\boldsymbol{x})\\&\text{subject to } \left[1 + \frac{\boldsymbol{h}(\boldsymbol{x})}{\mu}\right]^{\mu} \geq (1)^{\mu}\end{aligned} \quad (3)$$

Then the function ln(·) is applied to the inequality constraint result.

$$\begin{aligned}&\text{Minimize } f(\boldsymbol{x})\\&\text{subject to } \mu \ln \left[1 + \frac{\boldsymbol{h}(\boldsymbol{x})}{\mu}\right] \geq 0\end{aligned} \quad (4)$$

A Lagrangian is associated with the problem, so that the MBF is defined.

$$F(\boldsymbol{x}, \mu, \boldsymbol{u}) = \begin{cases} f(\boldsymbol{x}) - \boldsymbol{u}\mu \ \ln \left[1 + \dfrac{\boldsymbol{h}(\boldsymbol{x})}{\mu}\right] & \text{for} \quad \boldsymbol{h}(\boldsymbol{x}) > -\mu \\ \infty & \text{otherwise} \end{cases} \quad (5)$$

where $\boldsymbol{x}$ belongs to the interior of the relaxed feasible region, that is:

$$\left\{\boldsymbol{x} \in \Re^n | \boldsymbol{h}(\boldsymbol{x}) \geq -\mu\right\},$$

and $\boldsymbol{u} \in R^p$ are the non-negative estimates of the Lagrange multipliers in the optimal solution.

Polyak [28] introduced a novel approach to barrier methods for inequality-constrained problems. The MB method was introduced by expanding the barrier formulation to include a Lagrange multiplier sequence along with the barrier parameter sequence of the Logarithmic barrier method (Eq. (5)).

Applying the first-order necessary conditions to function (5), in relation to $\boldsymbol{x}$, with $\boldsymbol{u}$ and μ fixed, one obtains:

$$\nabla f(\boldsymbol{x}) - \frac{\boldsymbol{u}}{(\mu^{-1}\boldsymbol{h}(\boldsymbol{x}) + 1)} \nabla \boldsymbol{h}(\boldsymbol{x}) = 0 \quad (6)$$

Eq. (6), suggests the update of the estimates of the Lagrange multipliers, following the rule:

$$\boldsymbol{u}^{k+1} = \frac{\boldsymbol{u}^k \mu^{k+1}}{\boldsymbol{s}^{k+1} + \mu^{k+1}} \quad (7)$$

Vector $\boldsymbol{x}$ is updated using (8):

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k + \sigma \Delta \boldsymbol{x}, \quad (8)$$

where the step size $\sigma > 0$ is found through the rule of Goldstein–Armijo according to [28] and $\Delta \boldsymbol{x}$ is the search vector that can be calculated through Newton's method.

One of the difficulties found in the modified barrier method is the calculation of the step size, σ, as it is difficult to define a good stopping criterion for the unconstrained minimum at each step.

3. Predictor–corrector modified barrier approach

The proposed approach is based on the pure modified barrier approach (PMBA) and the predictor–corrector method (PCM). First, the PMBA will be developed and then the PCM will be added. In the PMBA, the bounded constraints are transformed into two inequalities. Slack variables are introduced, transforming these inequalities into equalities. The slack variables are relaxed and treated by the MBF, which results in the expansion of the feasible set of the original problem. A Lagrangian is associated with the problem. The first-order necessary conditions are applied to this function, generating a system of nonlinear equations, whose roots are calculated by Newton's method.

The optimal reactive power flow (ORPF) is a particular case of OPF, in which active controls are fixed and the control optimization is related only to the reactive power. The ORPF problem is a nonlinear programming problem, which can be represented as.

$$\begin{aligned}&\text{Minimize } f(\boldsymbol{x})\\&\text{subject to } \boldsymbol{g}(\boldsymbol{x}) = 0\\&\underline{\boldsymbol{h}} \leq \boldsymbol{h}(\boldsymbol{x}) \leq \overline{\boldsymbol{h}}\end{aligned} \quad (9)$$

where $\boldsymbol{x} \in R^n$ is the control and state variable vector, which represents the voltage magnitude, phase angles, LTC's taps and phase shifter's control angles. The objective function $f(\boldsymbol{x})$ is the real power loss in transmission. The vector of equality constraints function, $\boldsymbol{g}(\boldsymbol{x}) \in R^m$, where $m < n$, is the set of power flow equations. The inequality constraints $\boldsymbol{h}(\boldsymbol{x}) \in R^p$, with lower bound $\underline{\boldsymbol{h}}$ and upper bound $\overline{h}$, represent the functional constraints of the power flow, i.e., limits of magnitude voltage and LTC' taps, active and reactive power flows in the transmission lines and transformers, limits of reactive power injections for reactive control buses and active power injection for the slack bus. This is a typical nonlinear and nonconvex problem.

The ORPF problem can be solved by the PMBA, in which the positive slack variables are introduced to transform the inequality constraints into equality ones.

$$\begin{aligned}\text{Minimize} \quad & f(\boldsymbol{x})\\ \text{subject to} \quad & \boldsymbol{g}(\boldsymbol{x}) = 0\\ & \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s}_1 = \overline{\boldsymbol{h}}\\ & \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_2 = \underline{\boldsymbol{h}}\\ & \boldsymbol{s}_1 \geq 0\\ & \boldsymbol{s}_2 \geq 0\end{aligned} \quad (10)$$

where the slack vectors $\boldsymbol{s}_1 \in R^p$ and $\boldsymbol{s}_2 \in R^p$.

The non-negative conditions of problem (10) are relaxed by the barrier parameter, representing an expansion of the feasible region