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Eddy currents generated in a system of two cylindrical conductors by a transverse alternating magnetic field

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ABSTRACT

In the paper, using the Bubnov–Galerkin method coupled with the separation of variables method, the eddy currents in a system of two cylindrical conductors placed in a transverse homogeneous magnetic field sinusoidally varying with time are investigated. The power losses caused by these currents are also determined. By dint of computer calculations, the graphs of these losses are traced.

1. Introduction

A bibliography concerning fields and their integral parameters of a system consisting of two cylindrical conductors is mentioned in [1]. For instance in the case of a direct current, analysis was realized by means of the methods: of superposition [2], of images [3] and of the separation of variables [4,5]. Whereas for an alternating current considerations were made provided that the only one of conductors is thick—then an integral equation [6] and the method of separation of variables [7,8] were applied. It was caused by an additional difficulty, which appears for alternating fields namely, such that they fulfil complex Helmholtz's equation in conductors, for which the separation of variables is impossible in the bipolar coordinate system. However, the Bubnov–Galerkin method may be used for solving this equation in this system, which thereby afforded in [1] the possibility of analysing skin effect in a bifilar lead consisting of cylindrical conductors. This method has found a wide range of applications, about what one can convince oneself looking through the references list of [9–11]. Though the Bubnov–Galerkin method is approximative, nevertheless it allows to obtain the exact values of integral parameters provided that sequences of basic functions used to computations are complete both in the complex space of quadratically integrable functions and in the energetic space of the operator −laplacian. In this context, the principles of such sequences creation were given in appendices to [10,12], and in the papers [13,14] this subject-matter got treated more generally.

The aim of this paper constitutes analysis of the eddy currents generated in a system of solid cylindrical conductors by a transverse alternating homogeneous magnetic field with the help of the Bubnov–Galerkin method used inside the conductors and connected with the separation of variables method outside of them. Though the paper is theoretical, nevertheless cables with two strands of circular cross-section are universally used. Therefore, the extent of potential applications is wide. So the considered problem has a practical significance.

We shall leave out a description of the method whose full version is presented in [12,15] and its shortening in [1,10].

2. Description of the considered system

The cross-section of a system consisting of two solid cylindrical conductors placed in a transverse alternating uniform magnetic field with the complex r.m.s. induction $\mathbf{B}_0 = B_0\mathbf{1}_{\mathbf{B}_0}$ is shown in Fig. 1. The radius of the conductors amounts to r, and the distance between their geometric axes is equal to $d = 2w$. We denote by D_{I} and D_{II} the cross-sections of the conductors on the right-hand side and left-hand side of Fig. 1, respectively, and by D_{III} that of free space. The boundaries of D_{I} and D_{II} will be denoted with S_{I} and S_{II}. The considerations will be made in the right-handed bipolar system of coordinates (θ, η, z), $-\pi \le \theta \le \pi$, $-\infty \le \eta \le \infty$, which are connected with the rectangular coordinates as follows [16] (in [16] the bipolar coordinates are put in order (η, θ, z), which gives a left-handed system):

$$x = \frac{a \sinh \eta}{\cosh \eta - \cos \theta}, \quad y = \frac{a \sin \theta}{\cosh \eta - \cos \theta}, \quad z = z. \qquad (1)$$

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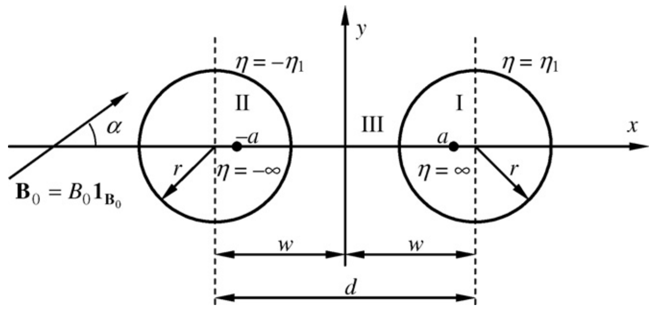

Fig. 1. Cross-section of a system of two cylindrical conductors.

Observe that for $\eta = \pm\infty$ we obtain the two poles $x = \pm a$, $y = 0$, which explains the name of the bipolar system. The dimensions r and w fulfil the following relations [16]:

$$r = \frac{a}{\sinh \eta_1}, \quad w = a \coth \eta_1, \tag{2}$$

from which it follows that

$$a = r(W^2 - 1)^{1/2}, \tag{3}$$

$$\exp(\pm\eta_1) = W \pm (W^2 - 1)^{1/2}, \tag{4}$$

where

$$W = \frac{w}{r}. \tag{5}$$

It is easy to verify that the only nonzero component, that is the z-component A_0, of the vector potential of the induction $\mathbf{B}_0$ is as follows:

$$A_0 = B_0^h y - B_0^v x = A_0^h + A_0^v, \tag{6}$$

where the horizontal component B_0^h and the vertical one B_0^v of the induction $\mathbf{B}_0$ amount to

$$B_0^h = B_0 \cos\alpha, \quad B_0^v = B_0 \sin\alpha. \tag{7}$$

The total (i.e. due to both the applied inducing field and the field generated by the currents induced in the conductors), complex r.m.s. magnetic induction $\mathbf{B}$ of the considered system may be presented as the sum of two vectors:

$$\mathbf{B} = \mathbf{B}^h + \mathbf{B}^v \tag{8}$$

where $\mathbf{B}^h$ is connected with the induction $B_0^h \mathbf{1}_x$ whose the vector potential z-component $A_0^h = B_0^h y$ (more precisely, $\mathbf{B}^h$ is the resulting induction due to both the applied induction $B_0^h \mathbf{1}_x$ and the induction generated by the eddy currents induced in the conductors by $B_0^h \mathbf{1}_x$), and $\mathbf{B}^v$ is connected with $B_0^v \mathbf{1}_y$ whose the vector potential z-component $A_0^v = -B_0^v x$ (more precisely, $\mathbf{B}^v$ is the resulting induction due to both the applied induction $B_0^v \mathbf{1}_y$ and the induction generated by the eddy currents induced in the conductors by $B_0^v \mathbf{1}_y$). If we keep in mind that the point $(-\theta, \eta)$ is the mirror reflexion of the point (θ, η) with respect to the x-axis and that $(\theta, -\eta)$ is the mirror reflexion of (θ, η) with respect to the y-axis, then an easy reasoning concerning runs of the magnetic induction lines leads to a conclusion that the η-components of $\mathbf{B}^h$ and $\mathbf{B}^v$ must fulfil the following conditions of symmetry:

$$B_\eta^h(-\theta, \eta) = B_\eta^h(\theta, \eta), \quad B_\eta^h(\theta, -\eta) = B_\eta^h(\theta, \eta), \tag{9}$$

$$B_\eta^v(-\theta, \eta) = -B_\eta^v(\theta, \eta), \quad B_\eta^v(\theta, -\eta) = -B_\eta^v(\theta, \eta) \tag{10}$$

3. Eddy currents in the considered system

3.1. Electromagnetic field

Writing the formula $\mathbf{B} = \mathrm{rot}(A\mathbf{1}_z)$ in the bipolar coordinate system, we obtain the components of the magnetic induction

$$B_\theta = \frac{1}{e_1}\frac{\partial A}{\partial \eta}, \quad B_\eta = -\frac{1}{e_1}\frac{\partial A}{\partial \theta}, \tag{11}$$

where

$$e_1 = \frac{a}{\cosh\eta - \cos\theta}. \tag{12}$$

Having regard to (8), the vector potential z-component may be presented as the sum of two elements

$$A = A^h + A^v, \tag{13}$$

where A^h and A^v are the vector potential z-components related to $\mathbf{B}^h$ and $\mathbf{B}^v$, respectively. Moreover, the vector potential z-components in each region D_{I}, D_{II} and D_{III} will be written with the region index.

The vector potential in D_{III} satisfies Laplace's equation whose general solution in the bipolar system of coordinates has the form [16]

$$A_{\mathrm{III}} = C_0'\eta + D_0' + \sum_{n=1}^{\infty}(C_n' \mathrm{e}^{-n\eta} + D_n' \mathrm{e}^{n\eta})(A_n' \cos n\theta + B_n' \sin n\theta). \tag{14}$$

As the total current in the both conductors is equal to 0, the integral of B_θ over a circumference $\eta = \text{const} \neq 0$, surrounding one of the conductors, gives $C_0' = 0$ by virtue of Ampère's circuital law. Moreover, the vector potential is definite with exactitude to the gradient, and consequently the constant D_0' may be left out. Whereas the vector potential z-component A_{III}^h should contain A_0^h, and A_{III}^v should contain A_0^v. Thus (6), (7) and the conditions of symmetry (9), (10) together with the second relation of (11) and (12) lead to the following formulas describing the vector potential z-components in D_{III}, i.e. for $|\eta| \le \eta_1$:

$$A_{\mathrm{III}}^h = B_0 r \cos\alpha \left(\frac{y}{r} + \sum_{n=1}^{N_h} A_n^h \cosh n\eta \sin n\theta\right), \tag{15}$$

$$A_{\mathrm{III}}^v = B_0 r \sin\alpha \left(-\frac{x}{r} + \sum_{n=1}^{N_v} A_n^v \sinh n\eta \cos n\theta\right). \tag{16}$$

The factor $B_0 r$ was introduced that the coefficients A_n^h and A_n^v might be dimensionless. Moreover, it will appear that they are independent of α because of the trigonometric functions written before brackets.

In order to determine the vector potential in D_{I}, let us take the following two sequences:

$$\{\mathrm{e}^{m(\eta_1-\eta)} \sin n\theta : m, n = 1, 2, \ldots\}, \tag{17}$$

$$\{\mathrm{e}^{(m+k_n)(\eta_1-\eta)} \cos n\theta : m, n = 0, 1, \ldots\}. \tag{18}$$

It may be proved that linear combinations of their terms allow to uniformly approximate complex functions continuous in D_{I} and odd with respect to θ (and consequently to y) using the sequence (17) and even ones using (18). A proof for the sequence (18) is given in [1] and for (17) is analogous. As anyone can see, k_n defined as follows:

$$k_n = \begin{cases} 0 & \text{for } n = 0 \\ 1 & \text{for } n \neq 0 \end{cases} \tag{19}$$

is inserted into (18) to exclude terms dependent only on θ, which would take on different values at the same point $x = a$, $y = 0$ obtained