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Analytical model and algorithm for tracing active power flow based on extended incidence matrix

Kaigui Xie[a,*], Jiaqi Zhou[a], Wenyuan Li[b]

[a] State Key Laboratory of Power Transmission Equipment & System Security and New Technology at Chongqing University, Chongqing 400044, PR China
[b] System Planning and Performance Assessment, BC Transmission Corporation, Vancouver, Canada

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ABSTRACT

This paper proposes an analytical model and algorithm for tracing power flow (TPF). The concept, construction approach and properties of extended incidence matrix (EIM) are developed. By using results of an AC or DC power flow solution from any off-line program or state estimation, the extended incidence matrix, generation and load power vectors, and distribution factor matrix are derived so that the analytical model of power transfers between generators and loads can be built. The major advantage of the proposed method is that the matrix theory is used to directly build the TPF model and no proportional sharing assumption on the flow distribution is needed. The method was tested using a 4-bus system, and the IEEE 30-bus and IEEE 14-bus power systems. The case studies indicate that the developed technique can be applied to any power system with or without loop flows.

1. Introduction

Tracing power flow (TPF) has been received more attention in recent years due to power industry restructuring. For power systems under the environment of deregulation and transmission open access, it is extremely important to calculate contributions of individual generators and loads to line flows, active power transfers between individual generators and loads, distributions of power losses and charges for the utilization of lines [1–18].

Many methods have been presented to solve the TPF problem [1–18]. The most popular approaches include topological generation distribution factors [1–4], nodal generation distribution factors [5] and factors based on the generator domains [6–8]. Ref. [6] presented an approach which is suitable for large-scale power systems. Based on the concept of extraction and contribution factor matrix, a method was suggested by using downstream and upstream tracing sequences [4]. By using the concept of loop flow coefficient and series theory, Ref. [9] proposed a TPF method based on graph theory which can be applied to networks with loop flows. Refs. [11–13] presented some models and algorithms for determining the contribution of individual generators and loads to power losses or energy losses. Evaluating contributions of generators to power flows in transmission lines is formulated as a multi objective optimization problem and performed using a parallel vector evaluated particle swarm optimization algorithm in Ref. [14]. Ref. [15] presented an approach to allocating the cost of a transmission network to its users based on the principle of *equivalent bilateral exchanges* (EBE). This approach is extended to deal with the problem of transmission cost allocation in a large network with multiple interconnected regions or systems in [16]. The rationale of the sharing principle and power flow tracing methods has been further discussed in [17] and [18]. In [17], the power flow tracing is formulated as a linear constrained optimization problem in which multiplicity of the solution space is explored using fairness criteria. In [18], the proportional sharing principle is rationalized using game theory to obtain a possible solution for cost allocation.

A main principle used to trace electricity flow in the published literatures so far is proportional sharing [1–9]. The principle described in [1–5,9] assumes that each outflow (a flow leaving a node) on a line is dependent only on the voltage gradient and impedance of the line, and that the contribution of each inflow (a line flow entering the node) to each outflow is in the same proportion as the inflow on each line divided by the total inflow of all lines at the node. The proportionality described in [6–8] assumes that if the proportion of the inflow which can be traced to generator i is x_i, then the proportion of the outflow which can be traced to generator i is also x_i. The method proposed in [10] is based on the assumption that the ratio of each bus load to the total system load is constant. All the principles can be easily utilized from an application point

* Corresponding author at: College of Electrical Engineering, Chongqing University, Chongqing 400044, PR China. Tel.: +86 23 65112740; fax: +86 23 65102434.
E-mail address: kaiguixie@yahoo.com.cn (K. Xie).

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of view. However, a disadvantage is the fact that the power sharing principle for power transfers between generators and loads is not based on a mathematically strict derivation.

This paper presents an analytical model for TPF based on the concept of extended incidence matrix (EIM). The proposed method does not need any assumption associated with the power sharing principle. It can handle any power system with or without loop flows and is flexible to start with an AC or DC power flow solution. The results of case studies indicate that the implicit distribution principle of the proposed method has the same effect as the explicit proportional principle in [1–5] for cases without loop flow whereas it can also deal with cases with loop flow using the same formulation and procedure.

2. Extended incidence matrix

2.1. Definition and construction of extended incidence matrix

In this paper, the following equivalence has been used for simplifying the problem:

The active and reactive power losses due to transmission line resistance, reactance and charging capacitance are moved to two terminal buses of the line and modeled as "equivalent loads". Therefore, active and reactive line power flows are constant along the line with a definite direction. Such a system is therefore equivalent to a lossless power network (LPN). This equivalence is more or less similar to the method described in Ref. [4] although losses have been considered in an alternate way. In fact, this equivalence has been also commonly recognized in other similar calculations.

According to Kirchhoff's first law, it is obvious that the total inflow at a bus equals the total outflow from the same bus in a LPN. The inflow here is defined as the sum of powers injected by sources and powers imported to a bus from other buses. The outflow is defined as the sum of powers extracted from a bus by loads and powers exported to other buses. An incidence matrix (IM) can be used to express incidence relations between buses and buses or between buses and branches in a power system. By extending the concept of IM, the following EIM is proposed for tracing power flow.

Denoting EIM of a LPN as $\mathbf{A}=(a_{ij})_{n\times n}$, active power in branch i–j from bus i to bus j as P_{ij} (>0) and total inflow at bus i as P_{Ti}, we define the following expression for elements of EIM:

$$a_{ij}=\begin{cases}-P_{ij} & \text{for } i\neq j \text{ and } P_{ij}>0\\ 0 & \text{for } i\neq j \text{ and } P_{ji}>0\\ P_{Ti} & \text{for } i=j\end{cases} \tag{1}$$

where $P_{Ti}=\sum_{\substack{k=1,k\neq i\\ P_{ki}>0}}^{n} P_{ki}+P_{Gi}$, and $i,j=1,2,\ldots,n$

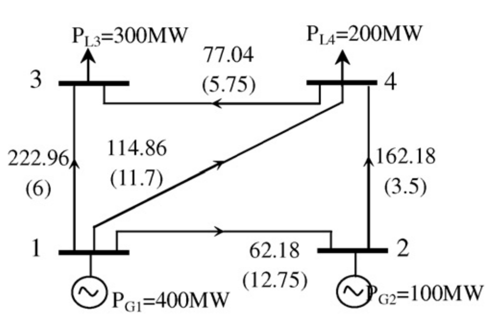

Fig. 1. The 4-bus lossless power system.

It can be seen from Eq. (1) that a_{ii} equals the total injecting active power at bus i, including the output of generators connected to bus i and inflows from other buses to bus i. The EIM of a LPN gives the information of bus–bus incidence relations, direction of active power in branch flows and total injecting power at a bus. Obviously, the EIM is an asymmetric matrix.

A 4-bus LPN [1] shown in Fig. 1 is used as an example to explain the proposed concepts. Based on Eq. (1), the EIM of the network shown in Fig. 1 is

$$\mathbf{A}=\begin{bmatrix}400 & -62.18 & -222.96 & -114.86\\ 0 & 162.18 & 0 & -162.18\\ 0 & 0 & 300 & 0\\ 0 & 0 & -77.04 & 277.04\end{bmatrix}_{4\times 4} \tag{2}$$

For example, $a_{22}=P_{12}+P_{G2}=62.18+100=162.18$.

2.2. Properties of extended incidence matrix

For the system shown in Fig. 1, the sum of all elements in every row of the EIM is $[0\,0\,300\,200]^T$, which equals the active load power vector, i.e., P_L. The fact can be extended to a power system with arbitrary number of buses.

Property 1. The sum of all elements in row k of an EIM equals the active load power at bus k. This property is mathematically expressed as:

$$\mathbf{A}E=P_L \tag{3}$$

For the system in Fig. 1, the sum of all elements in every column of the EIM is $[400\,100\,0\,0]$. This is the row vector of generation outputs, i.e., $(P_G)^T$. Similarly, in general, we have

Property 2. The sum of all elements in column k of an EIM equals the total active power of generators at bus k, i.e.,

$$E^T\mathbf{A}=(P_G)^T \tag{4}$$

or

$$\mathbf{A}^T E=P_G \tag{5}$$

There always exists at least one active power injection into each bus except for pure reactive power injection buses which are not taken into account here for active power distribution tracing. Therefore, the following Property 3 holds:

Property 3. Diagonal element of an EIM is $a_{ii}>0$.

Using the Property 2, we have

$$P_{Gj}=\sum_{i=1}^{n}a_{ij}=\sum_{i=1,i\neq j}^{n}a_{ij}+a_{jj}\geq 0 \tag{6}$$

Rewriting (6) yields

$$a_{jj}\geq-\sum_{i=1,i\neq j}^{n}a_{ij}=\sum_{i=1,i\neq j}^{n}|a_{ij}| \tag{7}$$

With the Properties 1–3, the following the Property 4 also holds:

Property 4. EIM of a LPN is a diagonally dominant and full rank matrix. In other words, an EIM is an invertible matrix.

For the network shown in Fig. 1, the inverse Matrix B of A is:

$$B=A^{-1}=\begin{bmatrix}0.0025 & 0.00095850 & 0.00226826 & 0.00159760\\ 0 & 0.00616599 & 0.00092694 & 0.00360959\\ 0 & 0 & 0.00333333 & 0\\ 0 & 0 & 0.00092694 & 0.00360959\end{bmatrix} \tag{8}$$