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Optimal placement of capacitors in a radial network using conic and mixed integer linear programming

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Abstract

This paper considers the problem of optimally placing fixed and switched type capacitors in a radial distribution network. The aim of this problem is to minimize the costs associated with capacitor banks, peak power, and energy losses whilst satisfying a pre-specified set of physical and technical constraints. The proposed solution is obtained using a two-phase approach. In phase-I, the problem is formulated as a conic program in which all nodes are candidates for placement of capacitor banks whose sizes are considered as continuous variables. A global solution of the phase-I problem is obtained using an interior-point based conic programming solver. Phase-II seeks a practical optimal solution by considering capacitor sizes as discrete variables. The problem in this phase is formulated as a mixed integer linear program based on minimizing the L1-norm of deviations from the phase-I state variable values. The solution to the phase-II problem is obtained using a mixed integer linear programming solver. The proposed method is validated via extensive comparisons with previously published results.

Keywords: Capacitor compensated distribution lines; Mixed integer linear programming; Nonlinear programming; Optimization methods; Reactive power control

1. Introduction

Reactive power flows in a radial distribution network always cause an increase in losses. At heavy loads, the losses due to reactive flows can become very significant. Moreover, these flows result in a line voltage drop that is greater than it would be at unity power factor. Consequently, capacitor banks are commonly installed on distribution lines to compensate for the customer reactive power requirement [1].

The problem of optimal capacitor placement on a radial network has been the subject of research for many decades thus resulting in a myriad of techniques [2], the most promising of which are based on optimization algorithms. Grainger at al. pioneered the application of non-linear programming methods to the problem of optimal capacitor placement [3]. In Ref. [3], both the capacitor locations and sizes were treated as continuous variables. Mixed integer non-linear programming methods were employed by Baran and Wu to select capacitor locations from a set of candidates [4]. Most recently, Aguiar and Cuervo [5] presented a mixed integer linear programming formulation that also accounts for the discrete capacitor sizes. The validity of this approach, however, strongly depends on the accuracy of the loss function approximation which has to be obtained via a properly chosen set of supporting hyperplanes [6]. The optimal capacitor placement problem in which capacitor sizes and locations can only take discrete values is believed to be non-deterministic polynomial (NP) complete [7], i.e., it is almost certain that solving for its global optimum cannot be done efficiently on a computer. According to many experts, a proof that a problem is NP complete is an adequate reason not to devote time and effort to trying to find a global solution [8]. Instead, it is recommended that one searches for a good near optimal solution of the problem. In fact, the published solution methods for the practical capacitor placement problem all adhere to this recommendation. The complexity of the problem has prompted many researchers to consider non-deterministic search techniques. Examples of such techniques which have been reported in the power systems literature include simulated annealing [9], genetic algorithms [10], tabu search [11], immune-based optimization [12], hybrid tabu search including heuristic features [7], hybrid micro-genetic algorithm in conjunction with fuzzy logic [13], ant

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direction hybrid differential evolution [14], a memtic evolutionary approach [15], and a variable scaling hybrid differential evolution method [16]. Although the above non-deterministic approaches have been validated on sample test systems, their success depends on tuning several algorithmic parameters. In general, these parameters are often system dependent and consequently their optimal setting requires skill on part of the user. A search algorithm which yields an optimal solution to the practical capacitor placement problem is therefore still sought after.

This paper proposes a deterministic two-phase approach to the optimal capacitor placement problem. Phase-I is based on conic programming. In fact, a recent paper by the author demonstrated that the radial load-flow problem could be efficiently solved using conic programming [17]. Phase-I extends the radial load-flow formulation by including candidate reactive sources at all nodes. The conic optimizer allocates reactive power to the sources with the aim of reducing the total system cost. In phase-I, the power allocated to each reactive source is treated as a continuous variable. Consequently, a computationally cheap global solution can be obtained using a path-following interior-point method. Phase-II takes into account the discrete nature of the sizes of the reactive power sources, i.e., the practical capacitor bank kVAr sizes. In essence, phase-II seeks a least absolute value solution having minimum deviation from the phase-I values of the state variables. The discrete nature of the capacitor sizes requires the least absolute value problem to be formulated as a mixed integer linear program, the solution for which can be obtained using branch-and-bound techniques. The main advantage of the phase-I/phase-II approach is that it can make use of existing high-powered software tools such as MOSEK [18] for solving conic and mixed integer linear programming problems. MOSEK [18] includes implementations of an interior-point method for conic programming and a branch-and-bound technique for mixed integer linear programming.

The rest of this paper is organized as follows. Section 2 reviews the formulation of the radial load-flow problem as a second-order cone program. Sections 3 and 4 discuss the phase-I and phase-II approaches, respectively. To simplify the presentation, one load level is initially considered. Section 5 extends the solution algorithm to account for load variations and capacitors of both switched and fixed types. Simulation results are reported in Section 6 and compared with solutions previously published in Refs. [5,7,10,11,13,15]. The paper is concluded in Section 7.

2. Radial load-flow

Consider a radial distribution network with one substation connected at node 0 and load nodes numbered from 1,. . .,N. It is assumed that the magnitude of the voltage at node 0 is specified. Denote by $\alpha(i)$ the set of nodes connected to node i and by P_{Li}/Q_{Li} the real/reactive power loads (in pu) at node i. Let the line model consist of the single-line equivalent circuit shown in Fig. 1 (all relevant quantities are in pu) and define $\theta_{ij}=\theta_i-\theta_j$, $u_i=V_i^2/\sqrt{2}$, $R_{ij}=V_iV_j\cos\theta_{ij}$ and $I_{ij}=V_iV_j\sin\theta_{ij}$.

It has been shown in Ref. [17] that the radial load flow solution can be obtained by solving the following second-order cone program:

$$\text{maximize} \sum_{ij\text{ lines}} R_{ij} \text{ subject to} \tag{1}$$

1. For $i=1,\ldots,N$:

$$-\sum_{j\in\alpha(i)} P_{ij} = -\sqrt{2}u_i \sum_{j\in\alpha(i)} G_{ij} + \sum_{j\in\alpha(i)} (G_{ij}R_{ij}-B_{ij}I_{ij}) = P_{Li}, \tag{2}$$

$$-\sum_{j\in\alpha(i)} Q_{ij} = -\sqrt{2}u_i \sum_{j\in\alpha(i)} B_{ij} + \sum_{j\in\alpha(i)} (B_{ij}R_{ij}+G_{ij}I_{ij}) = Q_{Li}, \tag{3}$$

$$u_i \geq 0 \quad \text{and} \quad u_0 = \frac{V_0^2}{\sqrt{2}}. \tag{4}$$

2. For all ij lines:

$$2u_iu_j \geq R_{ij}^2 + I_{ij}^2, \tag{5}$$

$$R_{ij} \geq 0. \tag{6}$$

The original variables (i.e. V_i and θ_i) can be easily deduced once the new adopted variables (i.e. u_i, R_{ij}, and I_{ij}) are computed. Note that the simple feeder model in Fig. 1 has been widely accepted for optimal capacitor placement. For instance, it has been used in Refs. [4,5,7,9–15].

3. Phase-I: conic programming

To simplify the presentation, one load level in a given period of time (e.g. 8760 h) is initially considered. The objective of the optimal capacitor placement problem is to minimize the costs associated with the installed capacitors, peak power, and energy losses. Because the radial distribution system has only one substation that injects power into node 0, the real power injected into node 0 is equal to the sum of the total real system losses and the total real load demand (which is constant). Consequently, minimizing the energy losses is equivalent to minimizing the energy injected at node 0. The objective can be therefore written as:

$$\text{minimize } K_c\sum_{i=1}^{N} Q_{ci} + (K_p + TK_e) \sum_{j\in\alpha(0)} P_{0j}, \tag{7}$$

where Q_{ci} is the reactive power injected by a capacitor at node i (in pu), P_{0j} is the real power flow from node 0 to node j (in

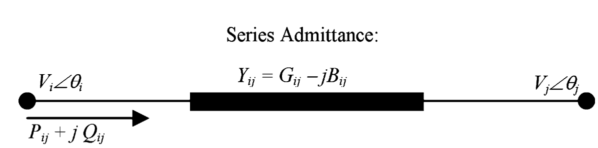

Fig. 1. Distribution line model.