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Impact of saturation nonlinearities/disturbances on the small-signal stability of power systems: An analytical approach

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Abstract

In this paper, a multi-objective optimization model is presented to estimate the practical stability region and maximum endurable disturbance rejection for a small-signal power system dynamic model with saturation nonlinearities and disturbance rejection. Iterative algorithms are developed to solve for Pareto optimized solutions (POS) of this optimization. Furthermore, as an application of this approach to power systems, a method to analyze the impact of saturation nonlinearities and disturbance rejection on power system small-signal stability is introduced based on the estimated stability region and maximum endurable disturbance rejection. Numerical results of a test power system with detailed saturated PSS controllers are described, indicating the reliability and simplicity of the method.

Keywords: Saturated systems; Power system stabilizer (PSS); Multi-objective optimization; Practical stability region; Golden section search (GSS) method; Iterative algorithm

1. Introduction

To improve the small-signal stability of power systems, much has relied on power system stabilizers (PSS) [1]. Thus, the issues of PSS parameter optimization [2,3] and control law design [4,5] are of interest. However, more often than not, the saturation nonlinearities, either intentionally designed or resulting from the limitations of equipments, are ubiquitous in the engineering fields [6], such as the power systems [7,8]. In general, PSS controllers are also subject to the saturation nonlinearities and disturbance rejection, which unavoidably affect the performance of PSS [9] and even can lead to loss of stability [6].

Therefore, if the saturation exists, the performance of the PSS control systems designed without considering saturation nonlinearities and disturbance rejection may seriously deteriorate [9]. Furthermore, the disturbance rejection may lead to the inexistence of stability region [10,11]. Utility engineers did look at the issue, mainly relying on extensive simulation studies [7,8]. Little attention has been paid to investigate the impact of such factors on system stability from analytical perspective.

The aim of this paper is to provide analytical methods to analyze the impact of saturation nonlinearities on power system small-signal stability when disturbance rejection exists, based on our recent work [9], where saturation nonlinearities are considered but the disturbance rejection is ignored. PSS performance study is taken as an example. The key is to characterize the stability region and maximum endurable disturbance rejection.

However, it is very difficult to handle the above-mentioned task today [6,10], since saturation nonlinearities make a simple linear system become a complex nonlinear system [10–12]. Therefore, many researches focus on the study of estimating stability region in recent years, e.g. [6,12] and the references therein, in which Hu derives a promising method to obtain an ellipsoid inside stability region by a quadratic Lyapunov function based on a convex LMI optimization [6]. This idea has been used in our recent work [9], and good results are obtained. Nevertheless, the disturbance rejection issue is not considered. In order to conquer this limitation, in some references, say [6,13], an invariant ellipse is derived as the practical stability region estimation, but an efficient algorithm is still lacking. In fact, in

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these papers, an auxiliary parameter is searched by a grid search mechanism, and the maximum endurable disturbance rejection is obtained by enumeration, so it unavoidably requires extensive computation burden.

To overcome this problem, we propose a multi-objective optimization model [14] to estimate the practical stability region and the maximum endurable disturbance rejection on the basis of [6,13]. Iterative algorithms are provided to solve for Pareto optimized solutions (POS) of this optimization, and some properties of these algorithms are proved also. Moreover, the procedures of the iterative algorithms are very simple and can be handled efficiently by the toolbox in Matlab.

The structure of the paper is as follows. In Section 2, the model of power systems with saturation nonlinearities and disturbance rejection is presented. Section 3 provides a multi-objective optimization model for estimating the practical stability region and maximum endurable disturbance rejection. The algorithms for solving for POS of the multi-object optimization problem are developed in Section 4. Based on the POS, a method to analyze the performance of PSS in power systems is provided in Section 5. In Section 6, a numerical example is described, indicating the reliability and simplicity of this approach. Section 7 draws the main conclusions of this work.

2. Power system model with saturation nonlinearities and disturbance rejection

Within a neighborhood around a given operating point, the ideal linear state space model of a power system can be expressed as [1,11]:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}'\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{w}(t); \quad \boldsymbol{x}_0 \in X_0,\ \boldsymbol{w}(t) \in W \tag{1}$$

where $\boldsymbol{x} \in R^n$ is the state; $\boldsymbol{u} \in R^m$ is the control; $\boldsymbol{A}' \in R^{n\times n}$ is the system matrix; $\boldsymbol{x}_0$ denotes the initial states; X_0 is the set of all initial states under consideration, $\boldsymbol{w}(t)$ denotes the disturbance rejection; $W \subset R^l$ is the set of all disturbance rejection under consideration and matrix $\boldsymbol{E}$ is the corresponding disturbance rejection matrix, respectively.

Due to actuator saturation which is considered to be a anti-windup function in this paper, a more realistic model is [6]:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}'\boldsymbol{x} + \boldsymbol{B}sat(\boldsymbol{u}) + \boldsymbol{E}\boldsymbol{w} \tag{2}$$

where sat(·) is a saturation function which is symmetric with respect to the origin, i.e.:

$$\text{sat}(\boldsymbol{u}) = [\text{sat}_1(u_1), \text{sat}_2(u_2), \ldots, \text{sat}_m(u_m)]^{\mathrm{T}},$$
$$\text{sat}_i(u_i) = \begin{cases} \bar{u}_i |u_i| > \bar{u}_i \\ u_i |u_i| \le \bar{u}_i \end{cases} \tag{3}$$

Thus, under a linear feedback control of form $\boldsymbol{u} = \boldsymbol{G}\boldsymbol{x}$, the closed loop system becomes

$$\dot{\boldsymbol{x}} = \boldsymbol{A}'\boldsymbol{x} + \boldsymbol{B}\text{sat}(\boldsymbol{G}\boldsymbol{x}) + \boldsymbol{E}\boldsymbol{w} \tag{4}$$

where pair $\{\boldsymbol{A}',\boldsymbol{B}\}$ is assumed to be controllable [11], $\boldsymbol{G} \in R^{m\times n}$ is the feedback gain matrix such that $Re(\lambda_i) < 0$ for all eigenvalues λ_i of matrix $\boldsymbol{A}' + \boldsymbol{B}\boldsymbol{G}$, and $Re(\lambda_i)$ denotes the real part of λ_i.

In our recent work [9], we did not consider the impact of disturbance rejection $\boldsymbol{w}(t)$ on the dynamic behaviors of the saturated system. In system (4), should disturbance rejection $\boldsymbol{w}$ be persistent, the origin is no longer an equilibrium point nor is it Lyapunov stable [10]. In this case, robust stability concepts such as uniform ultimate boundedness [10,15], also referred to as practical stability [11,16], can be applied to system (4). In particular, system (4) is said to be uniformly ultimately bounded (UUB) with respect to X_0 and W if, for all $\boldsymbol{x}_0 \in X_0$ and for every $\boldsymbol{w} \in W$, solution $\boldsymbol{x}(t)$ to Eq. (9) converges to a specified neighborhood around the origin. As such, the following region of practical stability is introduced for the subsequent investigation of system (4):

$$\Omega = \{\boldsymbol{x}_0 \in X_0 | \varphi_t(\boldsymbol{x}_0, \boldsymbol{w}) \text{ is UUB for every choice of } \boldsymbol{w} \in \boldsymbol{W}\} \tag{5}$$

where $\varphi_t(\boldsymbol{x}_0, \boldsymbol{w})$ denotes the trajectory of system (4) starting from the initial state $\boldsymbol{x}_0$. For simplicity, we make no difference between the terms "practical stability region" and "stability region" later.

From the definition of the stability region, the saturation nonlinearities result in that only the states in Ω can be considered to be stable. Furthermore, in order to analyze the impact of disturbance rejection on the dynamic behaviors of (4), we introduce a parameter, say α, for measuring the magnitude of disturbance rejection, i.e., we suppose that the disturbance rejection set W can be expressed as

$$W = \{\boldsymbol{w} \in R^l | \boldsymbol{w}^{\mathrm{T}}\boldsymbol{w} \le \alpha\} \tag{6}$$

Clearly, the relationship between W and α is

$$\alpha = \max_{\boldsymbol{w} \in W} \{\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}\} \tag{7}$$

To analyze the dynamic performance of system (4) later, we further consider set X_0 of expected initial states is a high-dimension ellipse defined as

$$X_0 = \{\boldsymbol{x} \in R^n | \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}_0\boldsymbol{x} \le \beta^2\} \tag{8}$$

where $\beta > 0$ is a variable to be decided later and $P_0 \in R^{n\times n}$ is a given and nonnegative definite symmetric matrix which is considered to be an identify matrix in the simulation. Namely, we assume that the expected initial states can be contained by an ellipse with fixed shape and variable size.

So from the previous analysis, the closed loop and asymptotically stable linear model:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A}' + \boldsymbol{B}\boldsymbol{G})\boldsymbol{x} + \boldsymbol{E}\boldsymbol{w} := \boldsymbol{A}\boldsymbol{x} + \boldsymbol{E}\boldsymbol{w} \tag{9}$$

is valid only for the states inside the polyhedron F, defined in the state space as

$$F = \{\boldsymbol{x} \in R^n | -\bar{\boldsymbol{u}} \le \boldsymbol{G}\boldsymbol{x} \le \bar{\boldsymbol{u}}\} \tag{10}$$

where the inequalities are based on "elements by elements", i.e., $F = \{\boldsymbol{x} | -\bar{\boldsymbol{u}}_i \le \boldsymbol{g}_i\boldsymbol{x} \le \bar{\boldsymbol{u}}_i, i = 1, 2, \ldots, m\}$ and g_i is the ith line of feedback gain matrix $\boldsymbol{G}$.