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Sympathetic inrush current phenomenon with loaded transformers

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ABSTRACT

The magnetization of a transformer is usually associated with the well-known phenomenon of inrush current. However, inrush current does not only affect the transformer being switched on. Instead, it has a significant impact also on all parallel connected transformers, which most certainly includes measurement transformers. This is known as a sympathetic inrush phenomenon. While the transformer being switched on might be a subject to a *sudden* high saturation level, the parallel transformers are *gradually* drawn to saturation as well, but of opposite polarity. The sympathetic inrush is expected in situations where the ohmic part represents a significant portion of the total system impedance. In contrast to other available literature on this subject, in this paper a modal approach to solving equivalent circuit by expressing differential equations in the state-equation form is selected. From the derivation of eigenvalues and eigenvectors, the phenomenon can be systematically investigated. A special attention will be given to circumstances, when the already operational transformer is fully loaded as within substations two or more transformers often operate in parallel, among which at least one of them is usually loaded. Finally, simulation results are compared to captured Wide Area Monitoring System (WAMS) measurements of the phenomenon and reasons for discrepancies are discussed.

1. Introduction

The transformer inrush phenomenon is very known and widely described for a long time. If the prospective magnetic flux (imaginative steady-state conditions prior to switching, describing the situation as if the switch was already on) in the transformer iron core at the moment of transformer energization differs from the value of zero, the flux DC component appears in the transformer core, which decays exponentially with time. Due to non-linear magnetizing characteristic of the transformer iron core (magnetic flux versus magnetizing current), a DC component of the magnetic flux can drive the transformer into saturation, in which the current drawn from the power system no longer changes linearly with the magnetic flux. Instead, each change in magnetic flux generates large currents that contain higher harmonic content. This usually happens during a transformer energization, especially in case of three-phase transformers, as the majority of circuit breakers perform a simultaneous switching of all three phase terminals. The intensity of the phenomenon can be significantly reduced or even omitted by controlling the moment of individual phase switching. However, not all transformers are equipped with mechanisms allowing such switching.

According to [1] transformer inrush currents might occur in different forms and can be divided into the following sub-categories:

- energization inrush (caused by re-application of voltage source to the transformer which has previously been de-energized. However, remanence could be still present)
- sympathetic inrush (caused by re-application of voltage source to the transformer, which operates in parallel to two or more other transformers)
- recovery inrush (caused by restoration of a voltage after clearance of a fault).

The subject of this paper is the second sub-category from the above list. Even though the first explanations of the phenomenon can be found in the quite early literature [2], the topic is also very relevant these days. Authors think it is reasonable to speculate that the need for additional explanation of such important phenomena always exist. This paper is based on findings and presentations, published in [1,3–9]. Certain additional explanations are added to papers already published with the strong support from clear graphical representation. Besides, in contrast to other available literature on this subject, in this paper a modal approach to solving equivalent circuit by expressing differential equations in the state-equation

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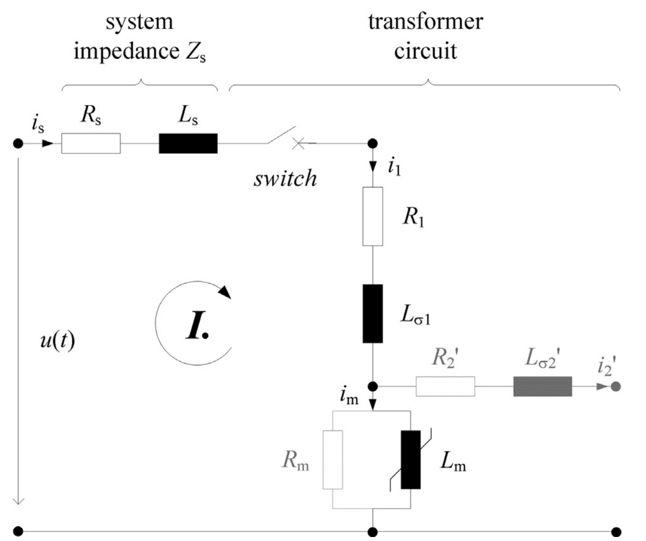

Fig. 1. Equivalent circuit of a power transformer, connected to a voltage source via system impedance.

form is selected. From the derivation of eigenvalues and eigenvectors, the phenomenon can be systematically investigated.

2. Energization inrush current

At first, it is reasonable to briefly explain the basic transformer energization inrush current. The most explanatory way to do so is by writing a voltage equation based on the Kirchhoff's second law for the transformer equivalent circuit, connected via system impedance $Z_s = R_s + j\omega L_s$ to a voltage source $u(t)$ (Fig. 1). Transformer equivalent circuit is usually thought of as a "T" circuit, where the denotations represent the following:

R_1 resistance of the transformer's primary winding,
$L_{\sigma 1}$ leakage inductance of the transformer's primary winding,
R'_2 resistance of the transformer's secondary winding (recalculated on the number of primary winding turns N_1),
L'_{s2} leakage inductance of the transformer's secondary winding (recalculated on the number of primary winding turns N_1),
R_m magnetizing resistance of the transformer, representing iron losses,
L_m magnetizing inductance of the transformer.

As a transformer inrush current occurs due to the non-linearity of L_m in the magnetizing branch, the simplest way to analyse the phenomenon is to assume that the transformer is un-loaded ($i'_2 = 0$, consequently the secondary branch is inactive and depicted in grey). This enables writing the following loop voltage equation, where the voltage source $u(t) = U_m \cdot \sin(\omega \cdot t + \alpha)$ is assumed ideally sinusoidal and the iron losses are neglected ($R_m \to \infty$, also depicted in grey):

$$-u(t) + [R_s + R_1] \cdot i(t) + \frac{d}{dt}([L_s + L_{\sigma 1} + L_m] \cdot i(t)) = 0 \qquad (1)$$

where $i(t) = i_s(t) = i_1(t) = i_m(t)$. For the purpose of simpler mathematical derivation, let us assume that L_m is linear (even though it is in fact the non-linearity of L_m the ground reason for the inrush phenomenon to be so important). Further, let us merge the transformer quantities and denote $R = R_1$ and $L = L_{\sigma 1} + L_m$. If we assume the relation between current and magnetic flux $L = \Phi / i$, the following expression can be obtained by rewriting (1):

$$\dot{\Phi}(t) = -\frac{R + R_s}{L + L_s} \cdot \Phi(t) + \frac{L}{L + L_s} \cdot u(t) \qquad (2)$$

What we are interested in is the passive circuit response, i.e. (2) without voltage source $u(t) = 0$. The so-called homogeneous solution of differential Eq. (2) with a constant C_1 is therefore more or less trivial and equals:

$$\Phi_{DC}(t) = C_1 \cdot e^{((R+R_s)/(L+L_s)) \cdot t} \qquad (3)$$

By taking into account that the switch from Fig. 1 is turned on at time $t = 0$ when flux equals $\Phi_{DC}(0) = \Phi_{DC0}$ (initial conditions), (3) becomes:

$$\Phi_{DC}(t) = \Phi_{DC0} \cdot e^{((R+R_s)/(L+L_s)) \cdot t} \qquad (4)$$

(4) represents the passive circuit response, which is obviously a DC component with a decaying rate determined by ratio between circuit serial ohmic resistance and serial inductance. Particular part of the differential equation solution on the other hand (AC flux component), which is not of main concern within this paper, can be obtained by using method of undetermined coefficient and some trigonometric laws. Assuming the solution in the form of:

$$\Phi_{AC}(t) = C_2 \cdot \sin(\omega t + \alpha) + C_3 \cdot \cos(\omega t + \alpha) \qquad (5)$$

its insertion into (2) gives the unknown constants:

$$C_2 = \frac{L \cdot U_m \cdot (R + R_s)}{(R + R_s)^2 + \omega^2 \cdot (L + L_s)^2}$$
$$C_3 = -\frac{\omega L \times U_m \cdot (L + L_s)}{(R + R_s)^2 + \omega^2 \cdot (L + L_s)^2} \qquad (6)$$

Considering trigonometric equalities:

$$\frac{1}{\sqrt{1+\beta^2}} = \cos\left(\arctan\left(\beta\right)\right)$$
$$\frac{\beta}{\sqrt{1+\beta^2}} = \sin\left(\arctan\left(\beta\right)\right) \qquad (7)$$

the particular solution equals:

$$\Phi_{AC}(t) = \frac{L \cdot U_m}{Z} \cdot \sin\left(\omega t + \alpha - \Theta\right) \qquad (8)$$

where

$$\Theta = \arctan\left(\frac{\omega \cdot (L + L_s)}{(R + R_s)}\right) \qquad (9)$$

$$Z = \sqrt{(R + R_s)^2 + \omega^2 (L + L_s)^2} \qquad (10)$$

It is clear that the voltage across transformer inductance $U_L(t)$ and its magnetic flux $\Phi(t) = \Phi_{DC}(t) + \Phi_{AC}(t)$ are $\pi/2$ shifted, as the definition dictates:

$$U_L(t) = N_1 \cdot \frac{d\Phi(t)}{dt} \qquad (11)$$

This is shown also in Fig. 2. By applying $u(t)$ at the moment when $U_L(t)$ is at its peak, $\Phi(t)$ would experience no DC component ($\Phi_{DC0} = 0$) as the prospective flux would at that time be equal to zero-dashed grey curve. However, by applying it at $U_L(t)$ zero-crossing when the prospective flux would be at its peak value, it would experience the worst-case DC component ($\Phi_{DC0} = \Phi_{DC0,max} = L \times U_m/Z$)—solid black curve. Of course, in above derivations no residual flux in the transformer core was assumed and consequently, Φ_{DC0} is in such case always between $-L \times U_m/Z \le \Phi_{DC0} \le L \times U_m/Z$. Also, due to impedance angle θ being very close to $\pi/2$ it is clear that $U_L(t)$ is almost in phase with $u(t)$.

As long as our goal is merely understanding the reason behind the flux DC component $\Phi_{DC}(t)$ occurrence, the simplification of considering L_m (or in above derivation L, as $L = L_{\sigma 1} + L_m$) linear is more or less irrelevant. However, as soon as inrush currents are concerned, one has to keep in mind and suitably consider that L_m