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A second order dynamic power flow model

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ABSTRACT

In this paper a novel second order power flow solution paradigm based on artificial dynamic models is proposed. The idea is to derive the second order optimality conditions for the power flow problem by reformulating the system equations into a set of ordinary differential equations, whose equilibrium points represent the problem solutions. Starting from the Lyapunov Theory we will demonstrate that the structure of this artificial dynamic model is stable with an exponential asymptotic convergence to the equilibrium points. The application of this technique for solving both unconstrained and constrained power flow problems is explained in details, and several numerical results are presented and discussed, demonstrating the effectiveness of the proposed methodology.

1. Introduction

Power flow analysis is one of the most fundamental and most heavily used tools in power system operation and planning. It has attracted a large amount of research efforts aimed at defining effective solution paradigms mainly based on Newton–Raphson (NR) [1] or fast-decoupled [2] algorithms. Broadly speaking, these approaches are based on the principle of linearizing the power flow equations and solving them by integrating iterative numerical algorithms and sparse triangular factorization techniques. Most of the studies reported in the literature reveal that these solution paradigms usually work quite well for solving the power flow under the condition of a well-defined Jacobian matrix but they could become instable when the attraction region of the initial solution guess is far away from the actual problem solution or when the power system is operating close to voltage collapse points. In the latter case the Jacobian matrix of the power flow equations becomes singular and the analyst is forced to heuristically determine which algorithm parameters should be tuned in order to resume the equations solvability [3,4].

Consequently much research efforts have been oriented in defining comprehensive solution paradigms aimed at addressing the power flow analysis even in ill-conditioned cases or at voltage collapse points [5].

In the light of this need, in papers [3,6] a non-divergent Newton power flow in rectangular coordinates using optimal multiplier theory is proposed. In [7] a robust continuation power flow method aimed at obtaining solution on any part of the P–V curve has been proposed.

To improve the convergence of these Newton based methods, the adoption of higher order approximation of the power flow equations has been proposed in [20–24]. These solution techniques aim at unifying the fields of nonlinear programming methods and Newton based methods by considering the second order term of the Taylor series approximation in polar or rectangular coordinates (namely the Hessian matrix). An interesting aspect of this paradigm is that an existing Newton based solution framework can be updated to a Hessian based program quite simply. In fact, the Hessian matrix of the power flow equations is somewhat less sparse than the corresponding Jacobian but enough so that sparse techniques could be effectively adopted [20]. Besides, it can be completely obtained from the Jacobian, thus avoiding extra explicit function evaluations in the solution algorithm [20], and its elements don't need not to be stored separately [25]. The studies reported in the literature show that, compared to the conventional Newton based methods, the second order power flow techniques require lesser iterations and exhibit better convergence characteristics [22]. Moreover, Hessian based paradigms allow the analyst to formulate the power flow problem by including additional system constraints so that the solution algorithm could solves both uncontrolled and controlled power flow problems.

A different perspective for solving power flow problems in the presence of matrix singularities has been proposed in [8,9]. The underlying principle is to formalize the power flow equations by a transient response of an equivalent dynamic system [8]. Although this paradigm was initially considered non-competitive compared

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to traditional Newton based methods [10], it has been recently re-evaluated and integrated into a Synthetic Dynamic Power Flow (SDPF) algorithm [9]. The latter aims at designing an artificial dynamic model whose equilibrium points correspond to the problem solutions. Starting from these important results in [11,12] a novel approach aimed at formulating the power flow problem on the vector continuous Newton's method is proposed. These papers rigorously demonstrate the formal analogy between the Newton based methods and a set of autonomous ordinary differential equations (ODEs). Thanks to this result, the power flow problem can be solved without requiring any matrix inversion which results particularly useful in the presence of infeasibilities or when the power system operates close to its loadability limits (where the Jacobian matrix could become singular). These operation points are not infrequent in modern power systems where the increasing load demand and the difficulty of upgrading the system infrastructures are pushing the power components to operate very close to their operational margins.

Despite these benefits, the application of dynamic power flow algorithms could reveal some shortcomings principally arising from the large convergence time required to simulate the state-variables trajectory of the artificial dynamic model.

As a consequence, the research for more effective methodologies aimed at improving the convergence performance of dynamic power flow algorithms is still an open problem and requires further investigations.

Armed with such a vision, in this paper we propose a novel dynamic formalisation of the power flow equations which takes advantage of the second order term of the Taylor series approximation in rectangular coordinates. The rationale is to enhance the mathematical kernel defined in [12,26] by integrating information on the Hessian matrix. We refer as "Hessian Dynamic Power Flow" the solution paradigm emerging from this mathematical model. According to this new dynamic paradigm, the power flow solutions can be obtained by computing the equilibrium states of an artificial dynamic system which is asymptotically stable. This allow us to overcome the inherent limitations characterising the iterative minimization algorithms that can fail to converge due to the highly nonlinearities of the first-order condition [12]. Besides the intrinsic architecture of the proposed dynamic model allows us to effectively solve the power flow equations also in the presence of uncertain, noisy or redundant data. We conjecture that these features could be particularly useful in power system state estimation. Moreover, compared to the dynamic power flow formalizations proposed in [12,26], we expect that the hessian dynamic power flow model exhibits faster dynamics and better convergence characteristics due to its ability to process higher order information.

In order to prove the effectiveness of the proposed computing paradigm, simulation results obtained on several IEEE test networks and on a real large-scale power system are presented and discussed.

The outline of the paper is as follows: Section 2 presents a brief review of the power flow problem. In Section 3, the theoretical basis of the proposed computing paradigm is analysed. The main features of the proposed approach are discussed in Section 4. To assess the effectiveness of the proposed methodology, detailed simulation studies for realistic systems are presented and discussed in Section 5. Finally, Section 6 summarizes the main conclusions and contributions of the paper.

2. Problem formulation

Power Flow Analysis deals with the steady state calculation of the voltage phasors at each network bus given a set of input parameters (i.e. load demand and real power generation) under certain assumptions (i.e. balanced system operation). Based on this information, the network operating conditions (i.e. real and reactive power flows on each branch, power losses and generator reactive power outputs), can be analytically determined.

The equations formalizing the power flow problem depend by the coordinate system adopted to represent the voltage phasors. In this paper we selected a rectangular coordinate system since it allows us to manage quadratic equations.

The power flow equations in rectangular coordinates include the real power balance equations at the generation and load buses, the reactive power balance at the load buses and the voltage magnitude at generation buses. These equations can be written as:

$$\begin{cases} P_i^{SP} = \sum_{j=1}^{N+1} \left(e_i e_j G_{ij} - e_i f_j B_{ij} + f_i f_j G_{ij} + f_i e_j B_{ij} \right) & i = 1, 2, \ldots, N \\ Q_i^{SP} = \sum_{j=1}^{N+1} \left(f_i e_j G_{ij} - f_i f_j B_{ij} - e_i f_j G_{ij} + e_i e_j B_{ij} \right) & i = 1, 2, \ldots, N_c \\ V_i^{SP,2} = e_i^2 + f_i^2 & i = N_c + 1, \ldots, N \end{cases} \tag{1}$$

where N_g is the number of PV buses, N_c is the number of PQ buses, $N+1$ is the slack bus $\left(N = N_g + N_c\right)$, V_i^{SP} is voltage magnitude specified at the ith, $\vec{V}_i = (e_i + jf_i)$ is the ith bus voltage phasor (in rectangular coordinates), $\vec{Y}_{ik} = (G_{ik} + jB_{ik})$ is the ikth element of the bus admittance matrix, P_i^{SP} and Q_k^{SP} are the real and reactive power injections specified at ith and kth bus, respectively.

Consequently, the overall problem requires the solution of a set of $2N$ nonlinear algebraic equations that can be grouped as follow:

$$\mathbf{s} = \mathbf{g}(\mathbf{x}) \tag{2}$$

where $\boldsymbol{x} = \left[e_1 \ldots e_i \ldots e_N \left| f_1 \ldots f_k \ldots f_N \right.\right]^T$ is the vector of unknown variables, $\boldsymbol{s} = \left[P_1^{SP} \ldots P_i^{SP} \ldots P_N^{SP}, Q_1^{SP} \ldots Q_k^{SP} \ldots Q_{N_C}^{SP}, V_{N_C+1}^{SP,2}, \ldots, V_j^{SP,2}, \ldots V_N^{SP,2}\right]^T$ is the vector of the specified active powers, reactive powers and voltage magnitudes, and $\mathbf{g} = \left[g_{p1} \ldots g_{pi} \ldots g_{pN}, g_{q1} \ldots g_{qk} \ldots g_{qN_C}, g_{vN_C+1} \ldots g_{vj} \ldots g_{vN}\right]^T$ is the vector of the power flow equations. The subscripts 'p', 'r' and 'v' represent active power, reactive power and voltage magnitude, respectively.

Due to the nonlinear nature of the power flow equations, the system (2) has not analytic solution and the power flow problem should be solved by identifying the state vector $\mathbf{x}$ which minimizes the following function [12,27]:

$$\begin{aligned} W(\mathbf{x}) &= \frac{1}{2}\sum_i (g_i(\mathbf{x}) - s_i)^2 = \frac{1}{2}(\mathbf{g}(\mathbf{x}) - \mathbf{s})^{\mathrm{T}}(\mathbf{g}(\mathbf{x}) - \mathbf{s}) = \\ &= \frac{1}{2}\mathbf{E}(\mathbf{x})^{\mathrm{T}}\mathbf{E}(\mathbf{x}) \end{aligned} \tag{3}$$

According to this paradigm the power flow problem can be solved by unconstrained minimization of a scalar positive semi-definite function.

In trying and addressing this issue the traditional solution approaches aim at solving the first-order derivative condition:

$$\frac{\mathrm{d}W(\boldsymbol{x})}{\mathrm{d}\boldsymbol{x}} = \mathbf{H}^T(\boldsymbol{x})\mathbf{E}(\mathbf{x}) = 0 \tag{4}$$

where $\mathbf{H}^T(\mathbf{x})$ is the Jacobian matrix (namely $H_{ik} = \frac{\partial g_i}{\partial x_k}$).

Solving (4) can be impracticable due to the nonlinear nature of the resulting set of equations, so numerical methods are employed in trying and obtaining a solution that is within an acceptable tolerance.