



# Enhancing the accuracy of rational function-based models using optimization



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## ABSTRACT

Rational function approximations are widely used for the modelling of frequency-dependent effects in power system components. This paper proposes to use optimization by the damped Gauss–Newton method to improve the accuracy of rational approximations calculated by three mainstream fitting techniques: Bode's asymptotic approximation, Sanathanan–Koerner iteration, and vector fitting. The optimization method is described in detail along with an outline of the numerical implementation. The proposed methodology is demonstrated for a synthetic example for the modelling of a measured reactor frequency response and for the rational fitting of the frequency-dependent parameters corresponding to a single-phase transmission line. In this example, we also present the impact of the optimization enhancement on the final time domain responses.

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## 1. Introduction

The modelling of power system components exhibiting frequency-dependent effects is usually performed in the frequency domain via curve fitting procedures, leading to rational function-based models. These models can be expressed in alternative form, including polynomial form, pole-zero form and pole-residue form. Such models are widely used in the modelling of overhead lines and underground cables, in power transformers at high frequencies and in frequency-dependent network equivalents. Rational fitting techniques have been developed since the 1950s for model synthesis based on frequency response data.

Levy [1] suggested in 1959 a process of linearized least squares (LS) based on a polynomial form representation of the rational function. In 1963, Sanathanan and Koerner [2] (SK) improved the Levy method by introducing iterations whereby the biasing in the approximation caused by the linearization was reduced by LS weighting. Rogers and Steiglitz [25] presented in 1967 an implementation of damped Gauss–Newton (DGN) method for the parameter identification of a transfer function when the output

contains independent Gaussian noise. This methodology is based on polynomial form by using the z-transform. In the implementation it is necessary to calculate filter parameters along with those of the model. Also, the input data for the transfer function should be known. Marti [3] presented in 1982 a method based on Bode's asymptotic approximation (Bode) for transmission line modelling, which quickly became popular for the modelling of overhead lines. Gustavsen and Semlyen [4] developed in 1999 a methodology known as vector fitting (VF), which, together with its different formulations [5–8], has become one of the most popular techniques. VF is related to SK but achieves superior performance by using pole-residue form instead of the polynomial form and implicit weighting by pole relocation instead of direct weighting. Furthermore, the poles are easily enforced to be stable. The biasing of VF was further improved [5,6] by introducing a relaxation in the formulation (RVF).

In addition to these three mainstream approaches, a number of other approaches have been proposed. Soysal and Semlyen [9] proposed in 1992 an improvement to the result by Levy via optimization by Gauss–Newton. Fernandes and Neves [10] demonstrated in 1997 the calculation of rational approximations using the Levenberg–Marquardt method. Noda [11] in 2005 presented an iterative algorithm that partitions the entire frequency range to avoid ill-conditioning of the system, which is based on the Levy method. More recently, Pordanjani et al. [12] in 2011 proposed a method based on a genetic algorithm.

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The aim of this paper is to enhance the accuracy of the rational approximation expressed in pole-zero form (Bode), polynomial form (SK) and pole-residue form (VF/RVF) by using the DGN method. SK, Bode and VF have been selected because they represent the mainstream fitting techniques.

In this work, DGN method is implemented in the Laplace domain for use with models on polynomial form, pole-zero form and pole-residue form.

In this paper, we first briefly review the appropriate physical constraints that a rational model must satisfy: stability, realness, symmetry and passivity. Then, we describe three rational fitting methods: Bode, SK and VF/RVF. We proceed with a description of DGN, which will be used to improve the accuracy by the models obtained with the aforementioned techniques. The procedure is demonstrated for the modelling of (1) a synthetic example, (2) a measured reactor frequency response, and (3) a transmission line. With the exception of VF/RVF, which is available in [13], all methods have been implemented in MATLAB.

## 2. Model constraints

A rational function-based model is required to comply with a number of constraints associated with the behaviour of physical systems:

- (1) *Stability*: The model must have all its poles in the left half-plane.
- (2) *Realness*: All model parameters must be real or come in complex conjugate pairs.
- (3) *Reciprocity*: In the case of multi-port systems, the state-space model may be required to be symmetrical with respect to its terminals, e.g., in the case of an admittance representation.
- (4) *Passivity*: The model cannot generate power whatever the terminal condition.

In this work, we pay attention only to constraints (1) and (2) because the reciprocity constraint does not apply to the scalar examples in this work and because the passivity constraint is normally assessed and enforced by perturbation in a post-processing step [14–17].

## 3. Fitting methods

In this section, we describe the main steps in the methods known as Bode, SK and VF/RVF while paying attention to the enforcement of the necessary constraints in Section 2.

### 3.1. Asymptotic approximation (Bode)

The asymptotic approximation known as Bode [18] was introduced in 1945 and afterwards applied by Marti [3] for transmission line modelling. Using Bode fitting leads to a pole-zero form given by

$$F(s) = k \frac{(s - z_1)(s - z_2) \cdots (s - z_n)}{(s - p_1)(s - p_2) \cdots (s - p_m)} \quad (1)$$

where poles and zeros are successively added to the total response function by tracing the resulting asymptote of the approximation to the magnitude function of the given response. To simplify the development of the technique, it is assumed [3] that the full set of poles and zeros are first order, real and distinct and lie in the left half-plane. It may also include a pole at the origin.

This method results in a model with magnitude function

$$|F(j\omega)| = \frac{K_0 \left| \frac{j\omega}{z_1} - 1 \right| \left| \frac{j\omega}{z_2} - 1 \right| \cdots \left| \frac{j\omega}{z_n} - 1 \right|}{|\omega| \left| \frac{j\omega}{p_1} - 1 \right| \left| \frac{j\omega}{p_2} - 1 \right| \cdots \left| \frac{j\omega}{p_m} - 1 \right|} \quad (2)$$

where  $K_0 = kz_1z_2 \cdots z_n/p_1p_2 \cdots p_m$  and  $s = j\omega$ . This equation can be expressed as

$$\begin{aligned} \log_{10} |F(j\omega)| &= \log_{10} |K_0| + \log_{10} \left| \frac{j\omega}{z_1} - 1 \right| + \\ &\cdots + \log_{10} \left| \frac{j\omega}{z_n} - 1 \right| - \log_{10} |\omega| \\ &- \log_{10} \left| \frac{j\omega}{p_1} - 1 \right| - \cdots - \log_{10} \left| \frac{j\omega}{p_m} - 1 \right|. \end{aligned} \quad (3)$$

In the implementation, one starts at the lowest frequency sample and compares the function to be fitted with the sum of the line segments given by the approximation of each term in (3), adding a pole or a zero when the asymptote departs from the target function by a tolerance value [3]. For transmission line modelling, one normally uses a relative tolerance criterion.

The realness criterion in Section 2 is enforced by restricting all poles and zeros to be real. The stability criterion is enforced by restricting the poles to lie in the left half-plane. It is also normal [3] to enforce the zeros to lie in the left half-plane by giving a minimum-phase shift function. In the implementation by Marti, the accuracy is enhanced by shifting the poles and zeros around their first asymptotes [19]. This feature is not used in the present work.

### 3.2. Sanathanan–Koerner (SK) iteration

The SK-method is based on the polynomial form:

$$F(s) \cong \frac{N(s)}{D(s)} = \frac{a_0 + a_1s + a_2s^2 + \cdots + a_ns^n}{1 + b_1s + b_2s^2 + \cdots + b_ms^m}. \quad (4)$$

The objective is to find all coefficients for the polynomials  $N(s)$  and  $D(s)$  in (4) that minimize the error function

$$\varepsilon(s) = F(s) - \frac{N(s)}{D(s)}. \quad (5)$$

As the problem (5) is non-linear in the denominator unknowns, Levy [1] multiplied (5) by  $D(s)$  to obtain a linear problem, thereby minimizing a weighted error function

$$\varepsilon'(s) = \varepsilon(s)D(s) = F(s)D(s) - N(s). \quad (6)$$

The unknown coefficients in (4) can now be solved by formulating (6) as an LS problem,  $\mathbf{Ax} = \mathbf{b}$ . The solution to the Levy problem is biased because of the multiplication of  $F(s)$  with  $D(s)$  in (6), which leads the weighting of the LS problem to be solved. To overcome this deficiency, Sanathanan and Koerner [2] proposed an iterative procedure where (6) is divided by the denominator from the previous iteration

$$\varepsilon''(s) = \frac{\varepsilon(s)D(s)_l}{D(s)_{l-1}} = \frac{F(s)D(s)_l}{D(s)_{l-1}} - \frac{N(s)_l}{D(s)_{l-1}} \quad (7)$$

The subscript  $l$  denotes the iteration number, and  $D(s)$  is considered 1 in the first iteration ( $l - 1 = 0$ ). Thus, (7) can be expressed as

$$\varepsilon''(s) = \frac{1}{D(s)_{l-1}} (F(s)D(s)_l - N(s)_l). \quad (8)$$

Eq. (8) is formulated as a weighted LS problem (WLS),  $\mathbf{W}\mathbf{Ax} = \mathbf{Wb}$ , where  $\mathbf{W}$  is the diagonal weighting matrix.

The realness criterion in Section 2 is enforced by restricting all coefficients for  $N(s)$  and  $D(s)$  to be real quantities, and stability is achieved by flipping any pole of  $D(s)$  in the right half-plane into the left half-plane by changing the sign of the real part and getting new coefficients for the polynomial.

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