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Modified routine for decreasing numeric oscillations at associations of lumped elements

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ABSTRACT

Some changes in the application of the numeric trapezoidal integration are analyzed for applications considering π circuits. It is considered numeric and computational proceedings for improving the numeric results obtained with associations of π circuits. In numeric integration solutions of the linear systems, it is common to represent these associations of π circuits by only one matrix. This representation introduces undesirable numeric oscillations in simulations of the dynamics of wave propagation in electrical systems. The proposed changes improve the results of application of cascades of π circuits associated to the trapezoidal integration, avoiding that the numerical oscillations, or Gibb's oscillations, have high values and are slowly damped. For the carried out simulations, different number of π circuits and voltage sources are checked, confirming the reduction of the influence of the numeric oscillations on the obtained results.

1. Introduction

Simulations and analyses of electromagnetic transients in power systems or electrical circuits are usually based on the matrix numeric applications [1]. They can be carried out using specific programs, such as the EMTP type programs [2,3] and PSpice software [4], as well as, digital mathematical tools [5–7]. These simulations and analyses can also be carried out using language programming for applying to numeric routines [8–11]. For these cases, the trapezoidal rule, or Heun's method, is usually applied to solve numeric integrations [1–12]. Considering these tools, the application of several lumped elements associated to the trapezoidal integration can lead to problems with numeric oscillations, or Gibbs' oscillations [1–7]. The application of associations of nominal π circuits has been considered computationally inefficient due to the use of a great quantity of π sections and the non-negligible influence of the numeric oscillations [1–7]. For electrical systems, as transmission lines, the frequency is an important influence on the determination of the electrical parameters [13–20]. This influence is not considered in the structure of nominal π circuits [1–5,13].

Decreasing Gibb's oscillations associated to the application of cascades of π circuits with trapezoidal rule, the influence of frequency on the transmission line parameters is not considered. The results with the influence of frequency can damp hardly the numeric oscillations and the major improvements of the proposed modified routine cannot be evidenced. So, in this paper, the transmission line parameters are intentionally restricted to frequency independent values [6,7]. Considering cascades of π circuits, the parameters of the lumped elements of these associations are included in numeric routines composing only one matrix. With this matrix, a linear system is obtained and solved using trapezoidal integration techniques [5–11,21]. The application of these cascades associated to the trapezoidal integration does not lead to efficient numeric routines. The order of the mentioned matrix depends on the number of π circuits. Applying n π circuits, it is necessary the $2n$-order matrix to represent them in the numeric routine. The increase of the matrix order is not directly related to proportional reductions on Gibb's oscillations [1,5,8–11,21]. Searching for better numeric representations for the cascade of π circuits, each infinitesimal π unit is modeled individually. The influences of adjacent π circuits are included considering the differential relations among the state variables: voltage on the capacitance and the current through the inductance [21,22]. The characteristics and the

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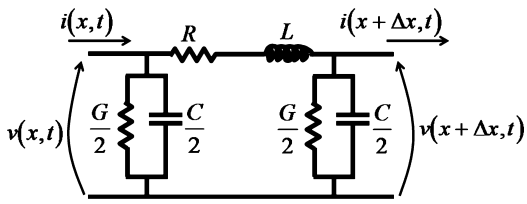

Fig. 1. An infinitesimal unit of π circuit.

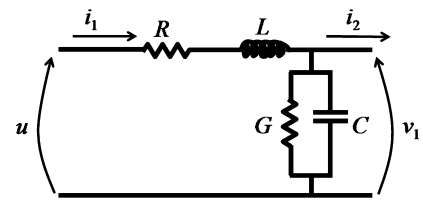

Fig. 2. A π circuit unit connected to a voltage source.

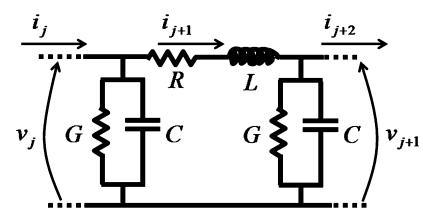

Fig. 3. An intermediate π circuit.

numeric stability of this routine are analyzed and Gibb's oscillations can be significantly decreased. The results are better than those obtained from other routines used previously [1,5]. Some investigations are carried out about the obtained results. It has based on eigenvalue and eigenvector analyses and it used tools of control theory. These investigations have not lead to detailed conclusions about the significant damping on the numeric oscillations in the simulated results, the numeric stability of the proposed routine, the limits of the application of this routine and the influence of the system parameters (time step, number of π circuits and line parameters). On the other hand, analyzes will be also carried out for fitting the influence of frequency in the proposed numeric routine [8–11,21,23]. Probably, the introduction of the frequency influence on the proposed routine will adapt it to analyses of more complex systems and circuits.

2. Mathematical bases

For generic applications, an infinitesimal π circuit is considered in Fig. 1. The infinitesimal influence of this element on voltage and current is represented by Δx. The complete circuit is represented by several units connected as a cascade [1–12,21]. Ideally, greater the number of π circuits, better the representation of the complete systems. A cascade of π circuits can be described as a linear system that can be solved using numeric integration methods:

$$\dot{x} = Ax + Bu \tag{1}$$

In (1), A and B are matrices and x and u are vectors. The A matrix represents the dynamics of the modeled circuit, the B matrix is related to the introduction of voltage and current sources in the circuit, the x vector is composed by the state variables and the u vector contains the voltage and current inputs. These elements compose a differential matrix relation. For numeric simulations, it used Heun's method for solving the linear system shown in (1). This numeric method is also called by the trapezoidal rule and it is more efficient than Euler's method considering the same time step [2,3,12].

Considering that the cascade of π circuits is opened at the sending end and receiving end terminals, the A matrix is [1]:

$$A = \begin{bmatrix} -\frac{G}{C} & -\frac{2}{C} & 0 & \cdots & \cdots & \cdots & 0 \\ \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} & 0 & \cdots & \cdots & \vdots \\ 0 & \frac{1}{C} & -\frac{G}{C} & -\frac{1}{C} & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} \\ 0 & \cdots & \cdots & \cdots & 0 & \frac{2}{C} & -\frac{G}{C} \end{bmatrix} \tag{2}$$

The A matrix's elements are determined from the parameters per length unit (R', L', G' and C') [1]:

$$R = R' \cdot \frac{d}{n} \quad L = L' \cdot \frac{d}{n} \quad G = G' \cdot \frac{d}{n} \quad C = C' \cdot \frac{d}{n} \tag{3}$$

In this case, d is the length of the modeled system or device and n is the number of π circuits. The state variables are the voltages at the capacitors and the currents through the inductors. For n π circuits, there are $2n+1$ state variables. The A and B matrices are $(2n+1)$-order ones. The x vector is composed by $2n+1$ state variables, where i_0 is related to a current source input. If a voltage source is applied, i_0 is not necessary and the matrices and vectors have $2n$-order.

$$x = \begin{bmatrix} i_0 & i_1 & v_1 & i_2 & v_2 & \cdots & i_n & v_n \end{bmatrix}^T \tag{4}$$

In (4), the index T identifies the transposed vector. Using trapezoidal rule, the system in (1) is changed into [1,5]:

$$x(k+1) = A_1 A_2\, x(k) + A_1 B_1 [u(k+1) + u(k)] \tag{5}$$

In (5), the elements are [1,5]:

$$A_1 = \left[I - \frac{\Delta t}{2} A\right]^{-1} \quad A_2 = \left[I + \frac{\Delta t}{2} A\right] \quad B_1 = \frac{\Delta t}{2} B \tag{6}$$

In (6), Δt is the time step, $x(k)$ is the known values of the state variables and $x(k+1)$ is the new values of the state variables calculated from $x(k)$ and based on t. The u vector has non-null elements for those points where the sources are connected to the system. The π circuits are composed by lumped parameters and the quantity of these elements should tend to the infinity, theoretically [13–20]. With the shown equations, simulations lead to results with numeric oscillations called Gibbs' ones.

3. Modified numeric routine [21]

In a cascade of π circuits, there are three different designs used for π circuits when a voltage source is connected to the sending end terminal. The first π circuit type is connected to the mentioned source and shown in Fig. 2. The π circuit type used between the both terminals is shown in Fig. 3. The receiving end terminal end is represented by Fig. 4.

For Fig. 2, considering the mesh voltages, it is obtained:

$$\frac{d i_1}{dt} = -\frac{R}{L} i_1 - \frac{1}{L} v_1 + \frac{1}{L} u \tag{7}$$

Using the node currents, it is obtained:

$$\frac{d v_1}{dt} = \frac{1}{C} i_1 - \frac{G}{C} v_1 - \frac{1}{C} i_2 \tag{8}$$