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Skin and proximity effects in the series-impedance of three-phase underground cables

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ABSTRACT

This paper is a contribution to the series-impedance computation of three-phase underground cables. The proximity and skin effects are considered in order to evaluate their influence on the resistance and inductance per unit length matrix elements.

The developed analytical methodology is based on the magnetic vector potential formulation where appropriate boundary conditions allow the magnetic field solution to be obtained.

Two cases are considered concerning the cable conductive sheath. First, an algorithm to calculate the series-impedance matrix of the perfect sheath case is implemented which is then used for the second case to calculate the series-impedance matrix of underground cables with an imperfect sheath.

Numerical results show that both skin and proximity effects lead to an increase in the resistance values and a decrease in the inductance ones.

1. Introduction

In urban and densely populated areas, where there is an abundant demand for electrical energy, underground cables are usually chosen for the transmission of electricity instead of overhead lines.

In order to guarantee quality and efficiency to the supplied electrical power, it is necessary to predict, simulate and optimize more and more accurately the constitutive parameters of underground cables, in particular the series-impedance matrix elements represented by the resistance and inductance per-unit-length elements. These parameters depend strongly on both skin and proximity effects.

The research topic of the paper, the magnetic field analysis of power transmission lines taking into account the skin and proximity effects, is not new. Many contributions have been published resorting to analytical approaches since the 1920s [1–5], including the contribution of the earth return path for underground cables [6–13]. More recently, numerical methods and techniques have been introduced and applied to generic configurations where heterogeneous, anisotropic or even nonlinear media are presented, in particular based on the Finite Element Method [14–17]. These works illustrate the current interest of the theme which remains up to date [1,4,14,17].

In this paper, a general analytical method is employed considering the following assumptions:

- A 2D field magnetic problem is considered. The underground cable is composed by parallel phase conductors placed inside a sheath, all of them solid cylindrical and traversed by axial currents.
- The soil is considered as a linear, isotropic and homogeneous conducting medium.
- The soil is characterized by a finite conductivity which allows the displacement currents to be ignored for the whole range of frequencies under concern (up to 1 MHz) – quasi-static approximation.
- The soil/air surface is taken as a plane parallel to the underground cable axis. The cable axis is placed at a depth h from the soil/air surface.
- Dielectrics are considered homogeneous, perfect and non-magnetic media.
- Conductors (phase conductors and sheath) are considered linear, isotropic and homogeneous media.

The analytical methodology is based on the magnetic vector potential formulation satisfying the fundamental field equations either in the dielectric or in the conducting media. Inside each

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conductor of the underground cable, the field solution is built by using a Bessel function development [1–4], for the field radial variation, regarding the skin effect and, by using a Fourier series development, for the field azimuthal variation, regarding the proximity effect.

In the dielectric media, a multi-pole description is used for the field [1,3,4] satisfying Laplace's equation. Series development coefficients are determined upon consideration of the appropriate boundary conditions on each conductor interface. The series impedance per unit length (p.u.l.) is then determined from the magnetic vector potential result as done in [1,6,13].

The adopted analytical method is of course limited to the cylindrical regular geometry in the presence of linear and sectional homogeneous media. However, it is powerful concerning the error control and the efficiency related to time and memory consumptions.

Two original contributions of this paper must be emphasized: a general phase conductor arrangement without any symmetry may be adopted for the power cable in despite that numerical applications are for triangular configurations; and, the development of a general solution putting together cylindrical solid conductors and hollow sheaths buried in the soil where skin and proximity effects are taken into account as a whole single inseparable phenomenon.

This paper is organized into four sections. Section 1 is introductory. Section 2 presents the series-impedance in sheathed cables where the vector potential solution is described in dielectric and conductive media, boundary conditions at conductor interfaces are imposed and the impedance matrix is presented for the cases with perfect and non-perfect sheaths. Section 3 is devoted to the presentation of numerical results for the phase conductor triangular configuration where skin and proximity effects are described and interpreted. Conclusions are in Section 4.

2. Series-impedance matrix in sheathed cables

Power underground cables constitute typical transmission line systems. They are characterized by the distributed series-impedance and shunt-admittance per unit length (p.u.l.) [17]. In this paper, focus will be given to the series-impedance matrix, $\left[\bar{Z}\right]$:

$$\left[\bar{Z}\right] = [R] + j\omega[L]\,[\Omega/\mathrm{m}] \tag{1}$$

where $[R]$ is the resistance matrix, $[L]$ is the inductance matrix and ω is the angular frequency.

An analytical approach based on the magnetic vector potential formulation is developed adopting the assumptions stated in Section 1. Currents flow in the axial direction, z, inside conductors and soil leading to a 2D magnetic field problem where the magnetic vector potential $\mathbf{A}$ is given by:

$$\mathbf{A} = A\vec{u}_z, \quad A = A(r,\varphi) \tag{2}$$

(r,φ) being the cylindrical transversal coordinates. Considering time harmonic fields, Maxwell equations for the quasi-static electromagnetic field lead to the following equations to be satisfied by the phasor of the axial component $\bar{A}$ of the magnetic vector potential:

$$\nabla^2\bar{A} = \begin{cases} j\omega\mu\sigma\bar{A} - \mu\sigma\bar{\eta} & \text{inside conductors} \\ 0, & \text{inside dielectric media} \end{cases}, \tag{3}$$

where μ and σ are the magnetic permeability and electric conductivity, respectively, and $\bar{\eta}$ is the voltage drop p.u.l. of the conductor given by $\bar{\eta} = -\mathrm{d}\bar{V}/\mathrm{d}z$, $\bar{V}$ being the phasor of the electric scalar potential of the conductor relative to the soil taken as the reference conductor. In these conditions, $\bar{\eta}$ is constant inside each conductor.

2.1. Magnetic vector potential inside dielectric media

The magnetic vector potential inside dielectric media satisfies the second equation of (3). The form of the solution may be obtained using the same rationale as indicated in [1], but in this case with a new contribution due to the presence of a cable conducting sheath. The global solution centered at the axis of the conductor k, O_k (Fig. 1(a)), which is obtained from the contributions of each of the N conductors and the sheath (conductor 0), all of them centered at O_k, may be obtained in a similar way as described in [1].

$$A^{(k)}(R,\phi) = \sum_{m=-\infty}^{+\infty} E_m^{(k)}(R)e^{jm\phi}, \quad R = \frac{r}{r_k} \tag{4}$$

where

$$E_0^{(k)}(R) = D_0 + C_0^{(k)}\ln\left(\frac{1}{R}\right) + \sum_{\substack{i=1\\i\neq k}}^{N} C_0^{(i)}\ln\left(\frac{r_i}{|\bar{w}_{ki}|}\right) + P_k \tag{5}$$

$$\begin{aligned} P_k = {} & \frac{1}{2}\sum_{p=1}^{+\infty}\left[D_pV_k(0,p) + D_{-p}V_k^*(0,p)\right] \\ & + \sum_{\substack{i=1\\i\neq k}}^{N}\frac{1}{2}\sum_{p=1}^{+\infty}\left[C_{-p}^{(i)}U_{ki}(0,p) + C_p^{(i)}U_{ki}^*(0,p)\right] \end{aligned} \tag{6}$$

$$E_m^{(k)}(R) = \frac{R^{|m|}}{2\,|m|}\left[\beta_m^{(k)} + |m|\,C_m^{(k)}R^{-2|m|} + \gamma_m^{(k)}\right] \quad m \neq 0 \tag{7}$$

$$\beta_m^{(k)} = \sum_{\substack{i=1\\i\neq k}}^{N} C_0^{(i)}\begin{Bmatrix} U_{ki}(m,0) \\ U_{ki}^*(-m,0) \end{Bmatrix} \tag{8}$$

$$\gamma_m^{(k)} = \sum_{\substack{i=1\\i\neq k}}^{N}\sum_{p=1}^{+\infty}\begin{Bmatrix} C_{-p}^{(i)}U_{ki}(m,p) \\ C_p^{(i)}U_{ki}^*(-m,p) \end{Bmatrix} + \sum_{p=|m|}^{+\infty}\begin{Bmatrix} D_pV_k(m,p) \\ D_{-p}V_k^*(-m,p) \end{Bmatrix} \tag{9}$$

where $U_{ki}(m,p)$ is given as indicated in (16) of [1]:

$$\begin{aligned} U_{ki}(m,p) &= \begin{cases} (-1)^m\left(\dfrac{\bar{w}_{ki}}{r_i}\right)^{-p}\left(\dfrac{\bar{w}_{ki}}{r_k}\right)^{-m}O(m,p) & \text{for } m \geq 0 \\ U_{ki}^*(-m,p) & \text{for } m < 0 \end{cases} \\ O(m,p) &= \begin{cases} \dfrac{(m+p-1)!}{(m-1)!(p-1)!} & \text{for } m > 0 \wedge p > 0 \\ 1 & \text{for } m = 0 \vee p = 0 \end{cases} \end{aligned} \tag{10}$$

$$\begin{aligned} V_k(m,p) &= \left(\frac{\bar{w}_k}{r_0}\right)^{p}\left(\frac{\bar{w}_k}{r_k}\right)^{-m}T(m,p) \quad \Leftarrow m \geq 0, \quad p \geq m \\ T(m,p) &= \begin{cases} \dfrac{p!}{(m-1)!(p-m)!} & \Leftarrow m = 0 \\ 1 & \Leftarrow m = 0 \end{cases} \end{aligned} \tag{11}$$

On the other hand, the global solution centered at the axis of the sheath O, which is obtained from the contributions of each of