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A quadratic approximation for the optimal power flow in power distribution systems

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ABSTRACT

This paper presents a quadratic approximation for the optimal power flow in power distributions systems. The proposed approach is based on a linearized load flow which is valid for power distribution systems including three-phase unbalanced operation. The main feature of the methodology is its simplicity. The accuracy of the proposed approximation is compared to the non-linear/non-convex formulation of the optimal power flow using different optimization solvers. The studies indicate the proposed approximation provides a very accurate solution for systems with a good voltage profile. Results over a set of 1000 randomly generated test power distribution systems demonstrate this solution can be considered for practical purposes in most of the cases. An analytical solution for the unconstrained problem is also developed. This solution can be used as an initialization point for a more precise formulation of the problem.

1. Introduction

Optimal power flow (OPF) is a classic problem for transmission system operation which has been extensively studied in the scientific literature [1–3]. The increasing penetration of renewable energies and the possibilities offered by communications in future smart-grids allow the use of OPF in power distribution systems [4,5] and especially in micro-grids [6–8].

OPF is a challenging problem due to the high number of non-convex constraints. Newton–Raphson, descendent gradient and interior points methods [9,10] are traditionally employed to obtain an optimal solution which may be the global optimum, although the problem may have several solutions that are locally optimal. These methods allow a decoupled formulations in the context of transmission networks, since nodal voltages are usually close to $1\angle 0$ and reactance/resistance ratio of transmission lines is frequently high. A good quality initial solution as well as a simple modeling which allows fast calculation of derivatives, are key features for a fast and accurate solution of the problem [11].

The problem is more challenging in power distribution systems due to the unbalanced operation and low X/R ratio of distribution lines. Hence, specially made algorithms are required. Non-linear programming as well as heuristic algorithms based on artificial intelligence have been proposed to find good solutions [12–14]. Evolutionary algorithms [15–17] and particle swarm optimization [18] have demonstrated to be efficient approaches for the problem. This type of algorithms allow an accurate modeling of the system by including constrains otherwise very difficult to consider. However, heuristic algorithms do not guarantee optimality and can be computationally cumbersome for real time operation.

Another approach for the problem in both, transmission and distribution networks, is the use of relaxations and simplifications in order to “convexify” the problem [19–23]. Semidefinite programming is one of the most promising modeling techniques for this propose [24,25]. The main advantage to reformulate a problem as a convex optimization problem is the capability to find global optimal solutions in an efficient way [26]. In addition, a convex formulation allows in some cases, the use of distributed methods. This is a key feature for future smart-grids.

The difficulty of the OPF lies in the non-convex nature of the load flow equations rather than in the number of variables. Different convex approximation have been proposed in the literature to address this problem. For example, in [27] a curve-fitting technique was used in order to linearize voltage-dependent load models. Other analytical approaches were presented in [28,29].

This paper introduces a quadratic convex approximation for the OPF in power distribution systems. This approximation is based on the linear formulation of the power flow presented in [29]. Different consideration are made ending at a non-iterative analytical solution for the relaxed problem. Both, the quadratic convex model and the analytical relaxed model are extensible to three-phase unbalanced

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distribution systems. These results have many potential applications including:

- As initial point for other non-linear or heuristic algorithms.
- As a practical solution in systems in which a close-to-the-optimal solution is acceptable.
- In markets regulation where a convex formulation is desired (i.e. the solution is unique and do not depend on the used algorithm).
- In real time operation where a fast solution is required.
- As part of another algorithm that requires to call many times an OPF as a sub-routine.
- As sensitivity analysis for power distribution systems.

Unlike conventional formulations, the proposed approximation uses complex voltages as state variables represented in rectangular form. Although similar formulations have been proposed before [30–33] the results presented here are different in three main aspects: first, the proposed formulation seeks an approximated model rather than an efficient implementation of a conventional Newton-based algorithm. This approximation has theoretical and practical applications from the power engineering stand point. Second, modeling and linearization is made entirely on complex variable before split in real and imaginary part for the optimization process. Off course, it might be possible first split and then linearize, but modeling in complex variables allows a straightforward extension to the three-phase case and inclusion of complex constrains. Third, an non-iterative solution is found for the relaxed case. Due to the non-convex characteristic of the problem, even the unconstrained case is difficult and can lead to local optimums [25]. Therefore, a global non-iterative solution is useful even for initialization purposes [34]. The model is applicable to transmission networks but is more suitable for power distribution systems where PV nodes are less common.

The remainder of this paper is organized as follows. Section 2 reviews the non-linear non-convex formulation of the OPF and analyzes the advantages of a rectangular formulation in power distribution systems. Section 3 presents the quadratic convex approximation of the OPF as well as a non-iterative analytical solution for the relaxed problem. In Section 4, the methodology is extended to three-phase unbalanced systems. Finally, Section 5 presents simulation results performed over an extensive set of test systems before Section 6 concludes.

2. Formulation of the OPF for power distribution systems

Different formulations for the OPF have been proposed in the scientific literature as a result of contributions from many researchers in this area. Two main formulations can be considered namely Polar-OPF and Rect-OPF, according to the representation of the state variables (polar or rectangular). Both formulations are equivalent. In the first case, decision variables are active and reactive power of distributed generators while voltages are state variables represented in polar form. In the second case, decision variables are currents injected by generators and state variables are voltages (both represented in rectangular form). Different objective functions can be considered including minimal generation costs, maximum market surplus and minimum losses, among others. In this paper, the minimum losses OPF is considered although the methodology can be extended for other objective functions.

Rect-OPF is less common in the literature than Polar-OPF [2]. However, it has some advantages in power distribution systems, especially in those cases where distributed generators are operated at constant power factor. In this formulation, voltages and currents are represented in rectangular form $((v_r, v_i)(i_r, i_i))$ as given in Eqs. (1)–(9)

$$\text{Minimize } P_L = \left(2\sum_{k=1}^{N} g_{(k0)} \cdot v_{r(k)} \cdot v_{(0)}\right) + \left(\sum_{k=1}^{N}\sum_{m=1}^{N} g_{(km)} \cdot v_{r(k)} \cdot v_{r(m)}\right) + \left(\sum_{k=1}^{N}\sum_{m=1}^{N} g_{(km)} \cdot v_{i(k)} \cdot v_{i(m)}\right) \quad (1)$$

subject to

$$i_{r(k)} = \sum_{m=1}^{N} g_{(km)} \cdot v_{r(m)} - b_{(km)} \cdot v_{i(m)} \quad (2)$$

$$i_{i(k)} = \sum_{m=1}^{N} g_{(km)} \cdot v_{i(m)} + b_{(km)} \cdot v_{r(m)} \quad (3)$$

$$v_{(k)}^2 \cdot i_{r(k)} = (p_{(k)} \cdot v_{(k)}^{\alpha_{(k)}} + P_{g(k)}) \cdot v_{r(k)} + (q_{(k)} \cdot v_{(k)}^{\alpha_{(k)}} + Q_{g(k)}) \cdot v_{i(k)} \quad (4)$$

$$v_{(k)}^2 \cdot i_{i(k)} = (p_{(k)} \cdot v_{(k)}^{\alpha_{(k)}} + P_{g(k)}) \cdot v_{i(k)} - (q_{(k)} \cdot v_{(k)}^{\alpha_k} + Q_{g(k)}) \cdot v_{r(k)} \quad (5)$$

$$v_{(k)}^2 = v_{r(k)}^2 + v_{i(k)}^2 \quad (6)$$

$$v_{\min} \le v_{(k)} \le v_{\max} \quad (7)$$

$$P_{\min} \le P_{g(k)} \le P_{\max} \quad (8)$$

$$Q_{\min} \le Q_{g(k)} \le Q_{\max} \quad (9)$$

where subscripts r, i represent real and imaginary part, and subscripts (k), (m) represent nodes (with (0) the slack node). Moreover, $g_{(km)}$, $b_{(km)}$ represent real and imaginary components of the nodal admittance matrix respectively, p and q are the nodal active and reactive power, P_g, Q_g are the active and reactive power delivered by distributed generators, and finally α is an exponent that represent the ZIP model of each load (i.e. 0 for constant power loads, 1 for constant current and 2 for constant impedance).

The main advantage of the Rect-OPF is that coupling between nodes are represented by a linear equation while the non-linear/non-convex equations are isolated to each bus. The main source of non-linear equations are constant power loads. Distributed generators can be considered as PQ buses for mathematical optimization modeling even if operated at constant voltage. Set point of the voltage can be calculated after the optimization is performed. On the other hand, most of the loads in distribution systems require a model which considers constant power, constant current and constant impedance loads (i.e. the ZIP model). Therefore, a linearization of the constant power loads is required in order to obtain a convex approximation.

3. Quadratic approximation

This section presents the quadratic approximation (Quad-OPF) from the Rect-OPF. The key step in this development is the linearization of the load flow equations which was first presented in [29]. For the sake of completeness, it is briefly presented below.

Let us consider a power distribution system whose topology is described by the nodal admittance matrix as follows:

$$\begin{pmatrix} \mathbb{I}_0 \\ \mathbb{I}_N \end{pmatrix} = \begin{pmatrix} \mathbb{Y}_{00} & \mathbb{Y}_{0N} \\ \mathbb{Y}_{N0} & \mathbb{Y}_{NN} \end{pmatrix} \cdot \begin{pmatrix} \mathbb{V}_0 \\ \mathbb{V}_N \end{pmatrix} \quad (10)$$

where 0 is the substation node (slack) and $N = \{1, 2, \ldots, n\}$ are the remained nodes. Along this section, a blackboard bold variable represents a complex matrix or vector while an unbold variable with sub index r or i represents its real or imaginary part respectively.