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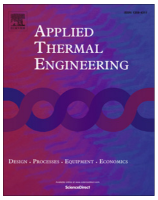

Research Paper

Thermal behavior of fluid within pipes based on discriminated dimensional analysis. An improved approach to universal curves

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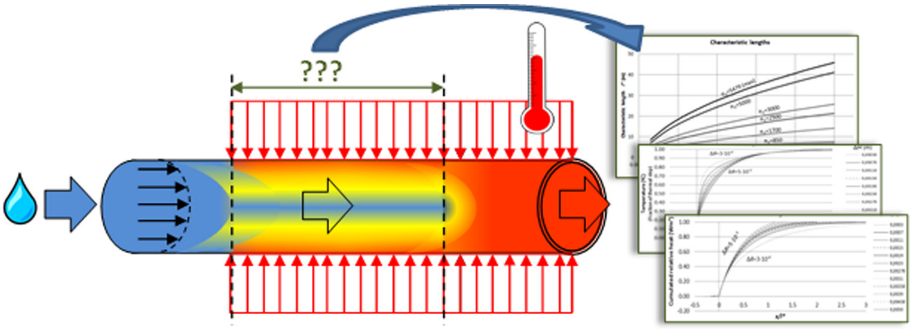
HIGHLIGHTS

- Discriminated dimensional analysis approaches a universal solution of Graetz Problem.
- A universal characteristic length abacus is offered; further pipe length is useless.
- Universal abacuses of temperature along the pipe is included.
- Universal abacuses of local and cumulated heat along the pipe is presented.
- Use of abacuses for design purposes is described.

GRAPHICAL ABSTRACT



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ABSTRACT

A new approach to the heat transmission problem concerning a fully developed laminar flux inside a pipe subject to an abrupt change of temperature in non-reactive conditions is presented. By means of dimensional discriminated analysis, representative dimensionless groups of 2D governing equations and boundary conditions of the problem are managed in order to find a set of dimensionless groups which describe the system's behavior. The characteristic length of the system, which plays a key role in the formulation, as well as the heat transfer and thermal distribution, are obtained numerically for the range of values of solar thermal flat-plate collectors using the network simulation method. The set of universal abacuses of temperature and heat based on the new dimensionless groups can be used in the design of industrial thermal devices and facilities.

1. Introduction

The thermal phenomena that take place in a fluid medium in the region of a pipe of infinite length where the temperature suddenly changes were first studied by Graetz in 1885 [1], although the thermal effect of the pipe wall on the fluid was not considered. This problem is of great interest for optimizing the design of pipes and ducts subject to thermal processes, and in which the flux can be modeled as a laminar phenomenon, as occurs in solar devices, oil pipelines, geothermal technology, heat exchangers, etc.

Since Graetz, the problem has been studied from several points of view. In 1980, Papoutsakis et al. [2] included axial conduction in the problem formulation, because this effect must be taken into account for low Péclet numbers in pipes. Then, in 1987, Cotta et al. [3] applied these results to parallel-plate channels. Ebadian

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Abbreviations: NSM, network simulation method; DDA, discriminated dimensional analysis; FPSC, flat-plate solar collector.

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Nomenclature

C_p	specific heat [J·kg^{-1}·K^{-1}]
k	thermal conductivity [W·m^{-1}·K^{-1}]
L	pipe length [m]
l^*	characteristic length [m]
R	inner radius of the pipe [m]
r	radial coordinate [ ]
t	temperature [°C]
t	time [s]
T_0	temperature of the fluid at the entrance and in the first zone of the pipe surface [°C]
T_1	temperature of the fluid at the exit and in the second zone of the pipe surface [°C]
u	velocity [m·s^{-1}]
z	axial coordinate [ ]
ΔR	pipe thickness [m]

Superscripts

$'$	dimensionless variable

Subscripts

ext	exterior
f	associated to fluid
m	medium
p	solid-liquid interface
r	associated to radius
s	associated to solid
z	associated to axis

Greek characters

π	monomial or dimensionless group
ρ	density [kg·m^{-3}]

and Zhang [4] solved the problem for constant heat conditions and variable temperatures through the pipe using the Fourier Transform. Şefik Bilir [5] contributed a research paper in 1992 about axial thermal conduction in a fluid, which was completed in 1995 [6] where axial fluid and wall conduction were considered (*conjugate extended Graetz problem*) and the bidimensional problem was solved using the finite difference method to speed up the process of obtaining the temperature distribution. In 2001, Silva Telles et al. [7] found an exact solution for flows inside a circular pipe, the annular space between pipes, and between parallel plates when the external temperature is variable, but not considering the pipes themselves. In 2003 Bilir and Ates carried out a parametric study of the thick-walled and two-regional pipe in the conditions of the Graetz problem, including transient process [8]. In 2004, Zueco, Alhama and González [9] studied bidimensional heat transmission in transient regime based on the effects of the wall thickness, the Péclet number, thermal conductivity at the fluid-wall interphase and thermal diffusivity.

Barletta [10] studied the effect of the viscosity of the fluid on the phenomena involved, concluding that the effect was negligible in most cases. In 2005, Peter Valkó [11] studied the viscous dissipation of a laminar flux in forced convection by means of the Laplace transform Galerkin method. Weigand and Gassned [12] obtained an entirely analytical solution to the conjugated extended Graetz problem in a parallel plate channel where the fluid is heated in a short length. In 2009 Cossali [13] established a solution to the problem of a flow subject to periodic variations inlet temperature in a pipe of any shape. Also in 2009 Onyejekwe [14] presented a green element formulation for solving the problem, including non-Newtonian fluids. Ates et al. [15] carried out a parametric study to analyze the effects of four parameters in a two region pipe with constant heat transfer. In 2015, Darici et al. [16] carried out a parametric study of the transient conjugate extended 2D heat transfer in thick pipes and minipipes subjected to a sudden change in ambient temperature; this work was completed in 2016 by Altun et al. [17] considering as boundary condition that the wall temperature varies periodically.

Although the thermal phenomenon of a fluid within a pipe has been widely studied in recent decades, referring to many specific cases and boundary conditions whose differential equations have been solved using different methods, researchers usually use classical dimensionless parameters, which provide some kind of generality to their results. Nevertheless, classical dimensional analysis has its own limitations. Therefore, in the works of Bilir [5,6] or Weigand and Gassned [12] the results are given in the form of a set of curves involving Nusselt (Nu), dimensionless temperature, Péclet (Pe) or other dimensionless numbers *vs.* different dimensionless parameters of axial length.

As an alternative, discriminated dimensional analysis (DDA) [18,19] focuses on the search for dimensionless variable groups that are physically consistent and which yield more general solutions, and introduces the characteristic length of the process – also considered a useful design variable – for this purpose. This new approach improves the outcome of the analysis of the thermal behavior of a fluid within pipes.

The concept of characteristic length is common in Physics [20–22], where it is used to define the scale of a physical system or process, especially when there are no suitable reference dimensions in a particular direction. In the present work it has a clear specific meaning, being understood as the length where the phenomena have been fully developed - which changes according to the geometric and thermal parameters involved. The characteristic length is also an important design tool and serves as indicator of the real useful length in the heat exchange process.

To contextualize this research with respect to previous works, Table 1 depicts the main methods applied for the theoretical analysis of thermal behavior of fluids within pipes.

The main difference in the application of DDA as opposed to CDA is that in the first the dimensionless groups or monomials are established following a well-structured method, which does not happen in CDA. As will be seen, DDA sometimes leads to monomials that have already been established in other works, but not on other occasions, or the number of monomials required for the definition of the problem is smaller: these are the difference between both approaches and the methodological improvement that DDA represents.

The aims of this paper are: (i) to apply DDA to study the thermal behavior of fluids in pipes, in order to yield a set of dimensionless parameters that better describe the problem, (ii), to obtain the characteristic length of the process and to describe its use, and (iii) to obtain a set of temperature and heat curves for different pipe thicknesses and Pe number as a function of the new dimensionless parameters based on the characteristic length and DDA.

The first two objectives are achieved by the analytical application of DDA to the 2D governing differential equations, and constitute the theoretical body of this work. The result is a much more precise formulation of the solution to the conjugate thermal fluid behavior within a pipe than that provided by classical dimension-