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Application of combined Newton–Raphson method to large load flow models

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ABSTRACT

Load flow is the most used calculations in power system operation planning. Renewable resources have caused that system operator has to do power flow analysis and/or contingency analysis as fast as possible in order to predict next step in power system control. LU Decomposition of Jacobian matrix remains the most computationally expensive task during Newton–Raphson iterative method. Computational time appears to be critical issue when load flow calculation is performed on large power system load flow models. In this case, Jacobian matrix LU decomposition should not be performed in iterations in which convergence rate is not violated but performed in iterations in which convergence rate drop below specified level. In other words, modified Newton–Raphson Method which eliminates the repeated Jacobian matrix LU decomposition and generic Newton–Raphson method are combined depending on convergence rate. The paper presents application of proposed combined Newton–Raphson method which is based on convergence rate control. Comparison of combined, Shamanskii, generic, and modified Newton–Raphson methods is carried out taking into consideration computational time and number of iterations required to achieve convergence of load flow models of various dimensions.

1. Introduction

For the time period when renewable resources did not have big influence on power system, several day-ahead forecast analyses during the day were enough for network operator to be ready in next 24 h. On the other hand, in accordance with stochastic nature of renewable resources such as wind, system operator should do the same analyses in intraday forecasts period between the day-ahead plan and the reality of the 5-min dispatch. Thus, computational request for the load flow calculation became very important.

The most widely used iterative procedure for AC load flow calculation is Newton–Raphson method including the methods that have been derived from this method. Load flow solution based on Newton–Raphson method has been defined at the late 1960 [1]. So far, a large number of various studies related to load flow solution have been presented. The studies were dealing with application of fixed-point techniques [2] and simplification of Newton–Raphson method in order to reduce time required for Jacobian update and LU decomposition, such as decoupled [3] and fast decoupled [4] Newton's methods. Enhanced decoupled load flow and simplified Newton–Raphson are also presented in [5] in order to minimize computational time. Modification of Newton–Raphson method where current injection mismatches are used for PQ nodes while power injection mismatches are used for PV nodes is presented in [6]. Comparison of decoupled load flow and Newton–Raphson method with constant matrices was carried out in [7]. In addition, Newton–Raphson method starting point [8], ill-conditioned and unsolvable cases [9–12] were also considered. Modification of Newton–Raphson method concerning generator reactive power limits was considered in [13].

The largest computational request in Newton–Raphson method is LU decomposition of Jacobian matrix. The main idea of this paper is to optimize number of Jacobian matrix LU decompositions during iterative procedure taking into consideration convergence rate and computational time. Convergence rate is checked during iterative procedure and if it drops below specified limit, Jacobian matrix is calculated and its LU decomposition is performed. Otherwise, if convergence rate remains higher than specified limit the same decomposed Jacobian matrix is used in forward–backward substitution step. In such a way convergence rate is controlled. Active convergence rate control, i.e. keeping convergence rate above specified limit during iteration procedure, is the novelty of the proposed combined Newton–Raphson method. The difference between the method proposed in this paper and Shamanskii

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method (or dishonest Newton–Raphson) [14] is that the proposed method controls convergence rate while Shamanskii method uses predefined number of variable updates by using the same factorized Jacobian matrix.

Implementation of Jacobian matrix incomplete threshold-based LU decomposition is considered in the paper since this technique has significant impact on computational time. Computational time and number of iterations depending on specified matrix element dropping threshold are analyzed. Investigation of incomplete LU decomposition calculation time was carried out in [15] as well. However, a technique of incomplete LU decomposition with predetermined number of fill-ins was considered in [15].

Comparative analysis is performed for combined, Shamanskii, generic (known also as: basic, full, complete and regular), and modified (known also as Newton's Chord or fixed slope) [16,17] Newton–Raphson methods taking into consideration load flow models of various dimensions.

Regional load flow models are made every day in purpose of Day Ahead Congestion Forecast (DACF) analysis by merging many national models. These DACF models are not well-conditioned containing many branches whose serial resistance is close to or even higher than serial reactance. When such branches exist in models Jacobian matrix items in upper right ($\partial P/\partial V$) and lower left rectangle ($\partial Q/\partial \theta$) cannot be neglected. That is the reason why decoupled and fast decoupled Newton–Raphson methods were not considered.

Jacobian matrix calculation and its LU decomposition are performed in each iteration if generic Newton–Raphson method is used. On the other hand, this is performed only in initial iteration if modified Newton–Raphson method is used. Combined Newton–Raphson method is compromise between these two Newton–Raphson methods. This means that Jacobian matrix calculation and LU decomposition are performed in initial iteration and in those iterations in which convergence rate drops below specified limit. This is the difference in comparison with the modification of Newton–Raphson method presented in [2] where switch from generic to modified Newton–Raphson method or vice versa is suggested to be done only once during iterative procedure. Previously analyzed computational aspects of Newton–Raphson method also suggested that Jacobian LU decomposition could be done in every two iterations rather than in each iteration, in order to save computational time [18].

Reduction of computational time of contingency analysis comprises some additional techniques, such as injection models of branches that avoid topology changes. However, such techniques are beyond the scope of this paper. The main purpose of the paper is to show how load flow computational time could be significantly reduced by controlling Newton–Raphson convergence rate and avoiding unnecessary LU decompositions of Jacobian matrix.

The paper presents combined Newton–Raphson method based on convergence rate control. The proposed method makes tradeoff between convergence and calculation speed. The method is straightforward but efficient and easy to implement.

The paper gives short preview of incomplete LU decomposition considering dropping threshold of matrix elements, fill-ins and round-off error in Section 2. Afterwards, proposed combined Newton–Raphson method and way of convergence rate control are presented in the following two sections. Comparative analysis of combined, Shamanskii, generic and modified Newton–Raphson methods taking into consideration load flow models of various system dimensions are given in Section 5.

Everyday DACF models are symmetrical so that investigation in this paper is limited to symmetrical transmission system models. No investigation has been carried out on unsymmetrical distribution systems containing unsymmetrical loads.

2. Jacobian incomplete LU decomposition

Theoretically, if zero dropping threshold for matrix elements is specified, LU decomposition is complete, i.e. none of matrix elements in matrix LU decomposition is neglected. However, due to finite number of figures representing floating point numbers in computer, LU decomposition is incomplete even if zero threshold is specified.

There are various threshold strategies and preconditioning techniques for incomplete matrix LU decomposition [19,20]. Implementation of incomplete matrix LU decomposition in the paper keeps all elements in the row that are higher than row norm multiplied by specified dropping threshold.

$$\tau_i = \tau \sum_{j=1}^{t} |a_{ij}| \tag{1}$$

where: t—number of elements in the row i; τ—specified dropping threshold; $|a_{ij}|$—absolute value of matrix element a_{ij}.

The only difference comparing to the incomplete LU decomposition with threshold defined in [19] is that all elements during LU decomposition whose absolute values are higher than product defined by Eq. (1) are kept rather than keeping certain number of the largest elements in the row. In such a way higher calculation precision is achieved which is important when matrices of large dimensions are decomposed. In other words, the round-off error is accumulated in each step of LU decomposition and in case of large matrix the error could increase resulting in invalid LU decomposition.

Concerning LU decomposition, the main point is to make compromise between required calculation precision and estimation time. Since LU decomposition is computationally the most expensive, computational time could be reduced by either using already decomposed matrix or performing LU decomposition with appropriate dropping threshold. Specified dropping threshold directly affects LU decomposition time, calculation accuracy, and number of Newton–Raphson iterations. Dropping threshold, required to achieve load flow convergence, depends on Jacobian matrix dimension and power system model conditioning. Specification of lower dropping threshold results in smaller numerical error of Jacobian matrix incomplete LU decomposition. On the other hand, it increases LU decomposition computational time. Besides dropping threshold, LU decomposition numerical round-off error depends on power system dimension, i.e. number of nodes and branches, electrical parameters of branches, Jacobian matrix preordering, etc.

There are various preconditioning techniques related to matrix reordering aiming to minimize number of fill-ins in lower and upper triangle of Jacobian matrix [21–24]. Minimization of number of fill-ins reduces both LU decomposition computational time and numerical round-off error. In this case slightly modified first Tinney's scheme [21] is used.

3. Combined Newton–Raphson method

The main idea of combined Newton–Raphson method is to keep convergence rate higher than specified limit which provides convergence of iterative procedure. If convergence rate drops below specified limit, Jacobian matrix is calculated and decomposed in order to improve convergence rate. Otherwise, the same lower and upper triangles of Jacobian that were used in iteration $k-1$ are used in iteration k. If convergence rate does not drop below specified limit during entire iterative procedure, Jacobian matrix is calculated and decomposed only in initial iteration. In such case combined Newton–Raphson method becomes equal to modified Newton–Raphson method.