Electric Power Systems Research 127 (2015) 160–164

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

# Error analysis of phase detector based on Clarke transform and arctangent function in polluted grids

Ignacio Carugati [a,b,*], Carlos M. Orallo [a,b], Sebastian Maestri [a,b], Patricio G. Donato [a,b], Daniel Carrica [a,b]

[a] Instituto de Investigaciones Científicas y Tecnológicas en Electrónica (ICYTE), Universidad Nacional de Mar del Plata (UNMdP), Juan B. Justo 4302, Mar del Plata, Argentina

[b] Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Av. Rivadavia 1917, Buenos Aires, Argentina

ARTICLE INFO

*Article history:*
Received 27 October 2014
Received in revised form 4 May 2015
Accepted 30 May 2015
Available online 19 June 2015

*Keywords:*
Three-phase systems
Synchronization
Phase detection
Clarke transform

ABSTRACT

Extracting phase information from a three-phase disturbed signal is a recurrent topic in power systems. The phase detector based on the Clarke transform and the arctangent function is a widely used technique to this end. However, the nonlinear nature of this method can derive in an error increment in the estimated phase angle when the input signal is distorted. These errors take the form of oscillating and constant terms and are not generally analyzed in the literature. Therefore, this work presents an error analysis of this phase detector that assesses its causes and effects. To do so, the disturbances over the estimated phase are modeled, the reduction of the phase detector performance is described, and the adverse effects produced by the absence of the pre-filter stage are discussed.



## 1. Introduction

Knowing the instantaneous phase angle of a signal is essential for the normal operation of many devices and applications, such as AC motors control [1–3], distributed generation systems [4–8] and power quality measurement systems [9]. This is because the precision of the method used for grid synchronization can affect systems performance significantly. For example, in power electronics converters related to distributed generation systems applications, a poor synchronization can result in an increased current harmonics injection and/or the injection of reactive power in an uncontrollable manner.

For single phase systems, phase detectors based on a single multiplier and classical Phase-locked Loops (PLLs) are widely used to obtain the instantaneous phase angle of the input signal [10,11]. On the other hand, for three-phase systems, a commonly used method for that task is the Clarke transform, which allows to represents the input signal in the stationary reference frame, and the subsequent calculation of the arctangent function. Among the applications of this phase detector in power systems are: synchronism systems [12–17] and sequence detectors [18–20].

When the three-phase signal is distorted, the estimated phase angle obtained by the Clarke transform and the arctangent function is distorted as well, leading to a poor performance of this phase detector. As a result, a common practice is to add filter stages to mitigate the disturbance effects, which can be implemented either before or after this method is applied. However, the limitations and restrictions of these solutions are not often analyzed.

On the other hand, although there are more efficient synchronization methods in the literature [7,13,17], the phase detector based on the Clarke transform and the arctangent function is a widely used technique due to its simplicity, mainly in industrial applications where a simple and well-known method is preferred over a more complex but efficient one.

As a result of the widespread use of this technique and the lack of a formal analysis of its performance under distorted operational conditions, in this work, the effects of disturbances on the estimated phase angle using the Clarke transform and arctangent function are modeled. The performance degradation of the phase detector is described, and the adverse effects produced by the absence of filtering before the phase detector are discussed. The generation of a DC phase error in the estimated phase angle is investigated, since this error cannot be mitigated by post-filtering stages, and so leads to deteriorated performance. Finally, this work provides an analysis tool to evaluate the convenience of use of the phase detector based

* Corresponding author at: Instituto de Investigaciones Científicas y Tecnológicas en Electrónica (ICYTE), Argentina. Tel.: +54 223 481 6600x254; fax: +54 223 481 0046.
*E-mail address:* icarugati@fi.mdp.edu.ar (I. Carugati).

http://dx.doi.org/10.1016/j.epsr.2015.05.027
0378-7796/

on the Clarke transform and arctangent function, by means of the computation of the phase error.

This paper is organized as follows. Section 2 reviews the operation principle of Clarke transform. The effect of input distortion in the phase detector based on Clarke transform and arctangent function is characterized in Section 3. In Section 4, some methods proposed in the literature, which use this technique for grid synchronization, are evaluated. Finally, Section 5 sets out the conclusions drawn.

## 2. Clarke transform

The Clarke transform is widely used in the literature to represent three-phase signals in the stationary reference frame [21]. In this way, the signal is modeled by a space vector whose two components preserve the initial phase, frequency and amplitude information with respect to the original system. As a result of the orthogonal relationship between its components, the instantaneous phase angle can be easily estimated by implementing an arctangent function.

This method assumes that the three-phase signal is composed of three ideal sinusoidal signals with equal initial phase, amplitude and frequency; and a constant phase difference of 120° between them. Under these conditions, this phase detector accurately estimates the instantaneous phase angle of a three-phase system.

However, when the three-phase signal is distorted by imbalances or harmonic components, the space vector components are also distorted, leading to poor performance of the phase detector. Under these operation conditions, the three-phase signal can be modeled in the stationary reference frame as:

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = V_{+1} \begin{bmatrix} \cos(\varphi_u(k)) \\ \sin(\varphi_u(k)) \end{bmatrix} + \sum_{\substack{n=-\infty \\ n \neq 0,1}}^{\infty} V_n \begin{bmatrix} \cos(n\varphi_u(k)+\varphi_n) \\ \sin(n\varphi_u(k)+\varphi_n) \end{bmatrix} \tag{1}$$

where $\varphi_u(k)$ is the system phase angle, $V_{+1}$ is the amplitude of the positive sequence fundamental component, $n$ identifies the $n$th harmonic, which can either be a positive ($n > 1$) or a negative ($n < 0$) sequence; and $V_n$ and $\varphi_n$ are the amplitude and initial phase of the $n$th harmonic, respectively. Note that the initial phase of the positive sequence fundamental component is equal to zero since it is adopted as the reference of the mathematical model.

Eq. (1) is the classical mathematical model of a stationary reference frame representation of a distorted three-phase signal. To obtain a more compact and convenient expression, $\varphi_u(k) - \varphi_u(k)$ is added to the argument of the second term of Eq. (1), resulting in:

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = V_{+1} \begin{bmatrix} \cos(\varphi_u(k)) \\ \sin(\varphi_u(k)) \end{bmatrix} + \sum_{\substack{n=-\infty \\ n \neq 0,1}}^{\infty} V_n \begin{bmatrix} \cos(n\varphi_u(k)+\varphi_n+\varphi_u(k)-\varphi_u(k)) \\ \sin(n\varphi_u(k)+\varphi_n+\varphi_u(k)-\varphi_u(k) \end{bmatrix} \tag{2}$$

Operating with the second term, this equation can be rewritten as:

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = V_{+1} \begin{bmatrix} \cos(\varphi_u(k)) \\ \sin(\varphi_u(k)) \end{bmatrix} + \sum_{\substack{n=-\infty \\ n \neq 0,1}}^{\infty} V_n \cos((n-1)\varphi_u(k)+\varphi_n) \begin{bmatrix} \cos(\varphi_u(k)) \\ \sin(\varphi_u(k)) \end{bmatrix} + \sum_{\substack{n=-\infty \\ n \neq 0,1}}^{\infty} V_n \sin((n-1)\varphi_u(k)+\varphi_n) \begin{bmatrix} -\sin(\varphi_u(k)) \\ \cos(\varphi_u(k)) \end{bmatrix} \tag{3}$$

and then:

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = V_\gamma \begin{bmatrix} \cos(\varphi_u(k)) \\ \sin(\varphi_u(k)) \end{bmatrix} + V_\delta \begin{bmatrix} -\sin(\varphi_u(k)) \\ \cos(\varphi_u(k)) \end{bmatrix} \tag{4}$$

where

$$\begin{cases} V_\gamma = V_{+1} + \displaystyle\sum_{\substack{n=-\infty \\ n \neq 0,1}}^{\infty} V_n \cos((n-1)\varphi_u(k)+\varphi_n) \\ V_\delta = \displaystyle\sum_{\substack{n=-\infty \\ n \neq 0,1}}^{\infty} V_n \sin((n-1)\varphi_u(k)+\varphi_n) \end{cases} \tag{5}$$

Finally, using the following trigonometric identity:

$$A\sin(x) + B\cos(x) = \sqrt{A^2+B^2}\cos(x - \tan^{-1}[A/B]) \tag{6}$$

and working with Eq. (4), it results in:

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \sqrt{V_\gamma^2 + V_\delta^2} \begin{bmatrix} \cos(\varphi_u(k) + \tan^{-1}[V_\delta/V_\gamma]) \\ \sin(\varphi_u(k) + \tan^{-1}[V_\delta/V_\gamma]) \end{bmatrix} \tag{7}$$

Eq. (7) proves that distortion in three-phase signals affects the amplitude and phase of space vector components. As a result, the extraction of the positive sequence component requires the mitigation of other components in an efficient manner.

## 3. Effect of distortion in phase detector

This section deals with the adverse effects produced by distortion in the estimated phase angle calculated by the phase detector based on the Clarke transform and the arctangent function.

### 3.1. Mathematical model of phase error

Assuming a three-phase signal represented in the stationary reference frame, the estimated phase angle ($\varphi_{est}$) can be calculated by:

$$\varphi_{est}(k) = \tan^{-1}\left[\frac{v_\beta(k)}{v_\alpha(k)}\right] \tag{8}$$

It is worth noticing that this calculus does not contemplate the fourth quadrant generated by $v_\alpha(k)$ and $v_\beta(k)$. Yet such consideration seems irrelevant for the present analysis. Replacing Eq. (7) in Eq. (8), the estimated phase results in:

$$\varphi_{est}(k) = \tan^{-1}\left[\frac{\sin(\varphi_u(k) + \tan^{-1}[V_\delta/V_\gamma])}{\cos\varphi_u(k) + \tan^{-1}[V_\delta/V_\gamma])}\right] \tag{9}$$