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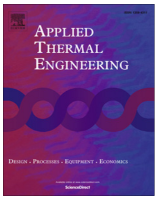

Research Paper

An inverse heat transfer method for predicting the thermal characteristics of a molten material reactor

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HIGHLIGHTS

- The solid/liquid phase change of the bank is investigated.
- The Levenberg-Marquardt-Method combined with the Broyden-Method is deployed.
- The inverse method is thoroughly tested for a wide range of test cases.
- The characteristics and the position of the temperature sensor are examined.

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ABSTRACT

An inverse heat transfer procedure is presented for predicting the time-varying thickness of the protective bank that covers the lining of the refractory brick walls of a molten material reactor. The inverse method predicts simultaneously influential thermo-physical properties of the reactor such as the thermal conductivity of the refractory brick wall, the thermal conductivity of the solid and of the liquid layers of the phase change material, and the time-varying reactor heat load. The inverse method rests on the Levenberg-Marquardt Method (LMM) combined with the Broyden method (BM). The effect (1) of the initial guesses for the unknown LMM polynomial parameters, (2) of the noise on the recorded temperature data, (3) of the location of the temperature sensors embedded into the brick wall and (4) of the number of recorded temperature data on the inverse predictions is investigated. Recommendations are then made concerning the installation and the operation of the temperature sensors.

1. Introduction

Molten material reactors, such as melting furnaces [1–4] and aluminum-electrolysis-cells [5–7] (Fig. 1) are used for material processing that requires high powers and elevated temperature. Their applications are in the production of aluminum and the smelting of materials such as copper, steel, and nickel calcine.

A common feature of these reactors is the formation of a bank that covers the lining of their brick walls. This bank is essential for protecting the inside surface of the brick wall from the highly corrosive molten material [1,2,8]. The bank is formed when the molten material comes into contact with the cooled wall and solidifies. Maintaining this bank and controlling its thickness while the furnace is operated, is a challenging task. The hostile environment that prevails inside the reactor forbids direct measurements of the bank with submerged probes. The problem is further complicated by the fact that the bank thickness is time-varying. It depends on the boundary conditions at the walls, the power load delivered to the furnace, and the thermo-physical properties of the refractory brick wall and of the molten material.

The alternative approach for handling this problem rests on inverse heat transfer techniques. In this approach, the protective bank is predicted by means of temperatures and/or heat fluxes recorded by sensors embedded into the refractory brick walls. The data gathered by the sensors are fed to an inverse heat transfer algorithm that deducts the thermal conditions that exist inside the furnace. From there, the thickness of the protective bank may be estimated.

Over the last decade, few investigations have been conducted with inverse heat transfer methods for predicting the time-varying bank inside molten material reactors. These inverse

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Nomenclature

C_p	specific heat [J/kg K]
dt	time step [s]
f	liquid fraction
h_∞	heat transfer coefficient [W/m^2 K]
I	total number of measurements
J	Jacobian matrix
k	thermal conductivity [W/m K]
L_{Brick}	width of the brick wall [m]
L_{PCM}	width of the PCM layer [m]
N	number of unknown parameters
$q''(t)$	heat flux [W/m^2]
$\vec{P}$	vector of unknown parameter
PCM	phase change material
$RRMSE$	relative root-mean-square errors [%]
$Error$	estimation errors [%]
$E(t)$	bank thickness [m]
t	time [s]
$\hat{T}$	estimated temperature [K]
W	weighting matrix
x	Cartesian spatial coordinate [m]
Y	measured temperature [K]

Greek symbols

ε	small number
μ	damping parameter
ρ	density [kg/m^3]
σ	standard deviation of the measurement error
$\sigma_{\hat{P}_j}$	standard deviation of the estimated parameter
Ψ	sum of squares norm
ξ	small number
δH	enthalpy [J/m^3]
Δ	difference
Ω^k	diagonal matrix
λ	heat of fusion [J/kg]
ω	random number

Subscripts

0	initial value
∞	ambient
Brick	brick wall
exact	exact solution
$E(t)$	bank thickness
F	freezing point
liq	liquidus
PCM, liquid	liquid (PCM)
max	maximum
P	parameter
PCM	phase change material
sol	solidus
PCM, solid	solid (PCM)

Superscripts

k	time iteration number
T	transposed matrix
^	estimated parameter
→	vector
↔	matrix

methods are based on various algorithms such as the conjugate gradient method with the adjoint equation [1,8–10], the Kalman Filter method (KF) [2,5–7,11,12] and the Levenberg–Marquardt method [13]. All these methods proceed in the same manner. They focus on the inverse prediction of the power load, i.e., the time-varying heat flux at ($x = L_{Brick} + L_{PCM}$) (Fig. 2). Once the heat flux is established, the protective bank thickness $E(t)$ is calculated using a direct method.

In all the aforementioned studies however, the thermal conductivities of the brick wall and of the PCM were fixed. In real situations, these conductivities are poorly known. For instance, the thermal conductivity of the brick wall may change as the bricks age. And the thermal conductivity of the phase change material (PCM) is often process dependent. As a result, the predicted time-varying bank thickness may be inaccurate when the wall and the PCM thermal conductivities are set in advance.

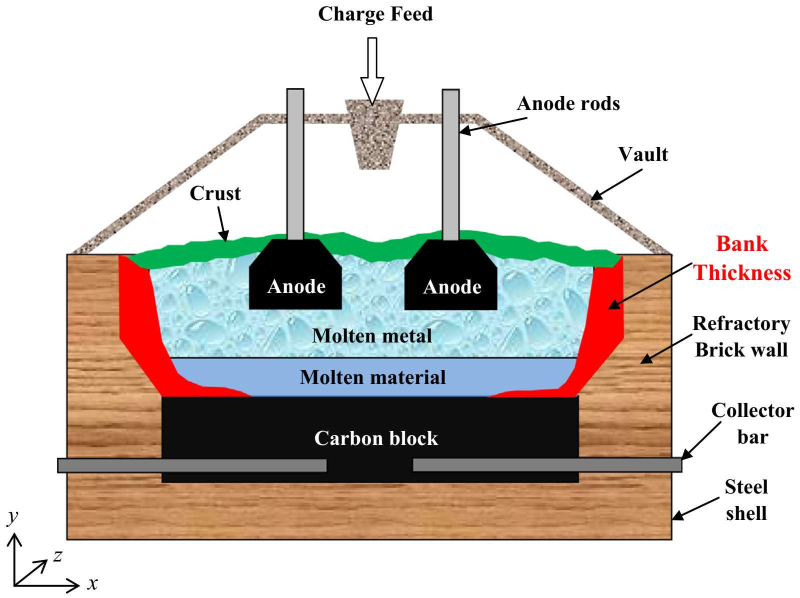

Fig. 1. Cross view of a typical molten material reactor. The thermal load is provided by the anodes.