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Discussion of the solvability of HVDC systems power flow with a sequential method

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ABSTRACT

This paper addresses the solvability of power flow for AC–DC system with sequential methods. As one of most wieldy used power flow analysis methods for AC–DC systems, this method solves the AC and HVDC systems iteratively. This paper shows that the solvability of the problem depends on a linearized coefficient matrix $\boldsymbol{G}$, which integrated the information of the network and HVDC control modes. To analyze the singularity of $\boldsymbol{G}$, all of the physically feasible combinations of the control modes are classified into seven categories. The conditions for the singularity of $\boldsymbol{G}$ are analyzed in detail. Systematical conditions for solvability of power flow with sequential methods for AC–DC systems are derived mathematically. Some numerical studies validate the proposed theoretical results.

1. Introduction

The power flow calculation methods for the AC–DC power systems can be generally classified into two categories [1–5]: the simultaneous methods and the sequential methods. Different from the simultaneous methods, the sequential methods are convenient to integrate the HVDC system calculation into existing AC power flow calculation programs with a little modification. As a result, various sequential methods have been proposed [6,7] and widely used. The pioneer work of sequential methods is based on the completely decoupled AC and HVDC subsystems [8–10]. Some improved sequential methods have been proposed by modifying the iteration matrix for the reactive power in the decoupled method or by modifying the Jacobian matrix for the Newton method [11]. Recent research [12] shows that the robustness of the HVDC subsystem's power flow calculation plays an important role in the convergence of the AC–DC system power flow calculation.

The linearized method might be the most widely used AC–DC power flow analysis method [13]. It is an extension of the 3-equation method for a 2-terminal HVDC system [2]. To use this method, the control modes and the converter characteristics are employed to construct the network equations first. By a series of mathematical transformations, a set of linear equations, i.e., $\boldsymbol{GX}=\boldsymbol{y}$, is derived. By solving these equations, the power flow can be solved.

Generally, the solvability of this method depends on the characteristics of $\boldsymbol{G}$. In this paper, the singularity of the coefficient matrix $\boldsymbol{G}$ for HVDC subsystems is analyzed in depth and the conditions of singularity are proved mathematically. Conditions for singularity of the coefficient matrix are proposed.

The remaining parts of this paper are organized as follows. Section 2 describes the models of HVDC systems for the linear method in AC–DC power flow calculation. Section 3 discusses the singularity of the matrix $\boldsymbol{G}$ and gives the theoretical proof. Section 4 shows the numerical results. Section 5 draws the conclusions of this paper.

2. Models of the HVDC system in linear method for power flow calculation

2.1. Characteristics of the converter

The schematic diagram for the connection between AC and HVDC subsystems is illustrated in Fig. 1.

For the AC–DC hybrid power systems, the joint buses which connecting the converter are the boundary of the AC and HVDC subsystems. The $\boldsymbol{V}_{ac}$ is the root mean square (RMS) value of line-to-line voltage of the joint bus. φ is the power factor angle. $\boldsymbol{P}_d$ is the active power with the positive direction shown in Fig. 1. $\boldsymbol{Q}_{\text{AC–HVDC}}$ is the reactive power exchanged between AC and HVDC subsystems. The $\boldsymbol{V}_{ac}$ and φ are calculated through AC subsystems and used in the HVDC subsystems. The $\boldsymbol{P}_{\mathrm{d}}$ and $\boldsymbol{Q}_{\text{AC–HVDC}}$ are calculated through HVDC system and then passed to the AC subsystems. The interface

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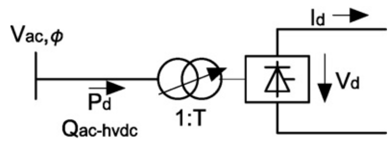

Fig. 1. The connection between AC and HVDC subsystems.

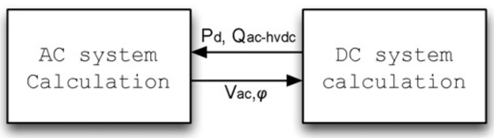

Fig. 2. The interface between AC and HVDC subsystems calculation.

between the calculation of AC and HVDC subsystems are shown in Fig. 2.

The mathematical model of the converter characteristics is:

$$
\begin{aligned}
&V_{d0} = \frac{3\sqrt{2}}{\pi} BTV_{ac} \\
&V_d = V_{d0}\,\cos\theta - \frac{3}{\pi} X_c I_d B \cdot \zeta \\
&\cos\varphi = \frac{kV_d}{V_{d0}} \cdot \zeta \\
&P_d = V_d I_d \\
&Q_{\mathrm{AC-HVDC}} = P_d \tan\varphi
\end{aligned}
\tag{1}
$$

where T is the converter transformer ratio, B is the number of converter bridges, $\boldsymbol{X}_c$ is the commutating reactance, $\boldsymbol{V}_d$ is the direct voltage, $\boldsymbol{V}_{d0}$ is the ideal no-load direct voltage, $\boldsymbol{I}_d$ is the direct current with the same positive direction as $\boldsymbol{P}_d$, $\boldsymbol{\theta}$ is the firing angle of the rectifier or the extinction angle of the inverter, ζ is 1 for rectifier and −1 for inverter, $k = 0.995$. k indicates the effect of the overlap angle. k would be 1 if the error caused by the overlap angle can be ignored.

2.2. Models of the controllers

The normal control modes for converters are: constant current control (CC), constant voltage control (CV), constant power control (CP) and constant firing angle (CA). According to the characteristics of converters and the various control modes, the models of the controllers for power flow calculation are [2]:

$$
\begin{cases}
V_{di} = V_{di}^{sp} & i \in \Omega_{CV} \\
I_{di} = \dfrac{P_{di}^{sp}}{V_{di}} & i \in \Omega_{CP} \\
I_{di} = I_{di}^{sp} & i \in \Omega_{CC} \\
I_{di} = \dfrac{\sqrt{2}V_{aci}T_i}{X_{ci} \cdot \zeta_i} \cdot \cos\,\theta_i^{sp} - \dfrac{\pi}{3X_{ci}B_i \cdot \zeta_i} \cdot V_{di} & i \in \Omega_{CA}
\end{cases}
\tag{2}
$$

where superscript sp means that the value is a preset value for the control mode, $i \in \Omega_{CV}$ represents that the ith converter is under the CV control mode, $i \in \Omega_{CP}$ represents that the ith converter is under the CP control mode, $i \in \Omega_{CC}$ represents that the ith converter is under the CC control mode, $i \in \Omega_{CA}$ represents that the ith converter is under the CA control mode.

2.3. Models of network equations of HVDC subsystems

The HVDC subsystem network equation is:

$$
\sum_j g_{dij} V_{dj} = I_{di} \tag{3}
$$

where g_{dij} is the correspond element in the node admittance matrix of the HVDC system. Substituting (3) with (2), we obtain,

$$
\begin{cases}
-I_{di} + \displaystyle\sum_{j \notin \Omega_{CV}} g_{dij} V_{dj} = -\sum_{j \in \Omega_{CV}} g_{dij} V_{dj}^{sp} & i \in \Omega_{CV} \\
\displaystyle\sum_{j \notin \Omega_{CV}} g_{dij} V_{dj} = P_{di}^{sp}/V_{di} - \sum_{j \in \Omega_{CV}} g_{dij} V_{dj}^{sp} & i \in \Omega_{CP} \\
\displaystyle\sum_{j \notin \Omega_{CV}} g_{dij} V_{dj} = I_{di}^{sp} - \sum_{j \in \Omega_{CV}} g_{dij} V_{dj}^{sp} & i \in \Omega_{CC} \\
\displaystyle\sum_{j \notin \Omega_{CV}, j \neq i} g_{dij} V_{dj} + \left(g_{dii} + \frac{\pi}{3X_{ci}B_i \cdot \zeta_i} \right) V_{di} = \frac{\sqrt{2}E_{aci}T_i}{X_{ci} \cdot \zeta_i} \cdot \cos\,\theta_i^{sp} - & \\
\displaystyle\sum_{j \in \Omega_{CV}} g_{dij} V_{dj}^{sp} & i \in \Omega_{CA}
\end{cases}
\tag{4}
$$

The matrix form is [3]:

$$
\begin{bmatrix}
-I & G_{VP} & G_{VI} & G_{VA} \\
0 & G_{PP} & G_{PI} & G_{PA} \\
0 & G_{IP} & G_{II} & G_{IA} \\
0 & G_{AP} & G_{AI} & G'_{AA}
\end{bmatrix}
\begin{bmatrix}
I_{dV} \\ V_{dP} \\ V_{dI} \\ V_{dA}
\end{bmatrix}
=
\begin{bmatrix}
-G_{VV} V_{dV}^{SP} \\
U_{dP}^{-1} P_{dP}^{SP} - G_{PV} V_{dV}^{SP} \\
I_{dI}^{SP} - G_{IV} V_{dV}^{SP} \\
DA_{dA}^{SP} - G_{AV} V_{dV}^{SP}
\end{bmatrix}
\tag{5}
$$

Rewritten it in a compact form as

$$
\boldsymbol{G}\boldsymbol{X}_{dc} = y_{dc} \tag{6}
$$

where matrix $\boldsymbol{I}$ is the identity matrix, subscript V, P, I and A indicate control mode as CV, CP, CC and CA, respectively, $\boldsymbol{G}_{VP}$ means the admittance sub-matrix related to those converters with CV or CP controller and the other sub-matrices are similar, $\boldsymbol{V}_{dV}^{SP}$ is a vector consisting of $\boldsymbol{V}_{di}^{sp}$, $i \in \Omega_{CV}$. $\boldsymbol{P}_{dP}^{SP}$ is a vector consisting of $\boldsymbol{P}_{di}^{sp}$, $i \in \Omega_{CP}$, $\boldsymbol{I}_{dI}^{SP}$ is a vector consisting of $\boldsymbol{I}_{di}^{sp}$, $i \in \Omega_{CC}$, $\boldsymbol{A}_{dA}^{SP}$ is a vector consisting of $\cos\theta_{di}^{sp}$, $i \in \Omega_{CIA(CEA)}$, $\boldsymbol{U}_{dP} = diag\,\{V_{dPi}, i \in \Omega_{CP}\}$. And,

$$
D = diag \left\{ \frac{\sqrt{2}E_{aci}T_i}{X_{ci} \cdot \zeta_i} \right\}, i \in \Omega_{CIA(CEA)} \tag{7}
$$

$$
\boldsymbol{G}'_{AA} = \boldsymbol{G}_{AA} + \boldsymbol{S}, \qquad \boldsymbol{S} = diag \left\{ \frac{\pi}{(3X_{ci}B_i) \cdot \zeta_i} \right\} \tag{8}
$$

If there is no CP controller, (6) is linear. $\boldsymbol{X}_{dc}$ can be solved directly by

$$
\boldsymbol{X}_{dc} = \boldsymbol{G}^{-1} y_{dc} \tag{9}
$$

Due to the U_{dP}, under CP control modes, the right vector of (5) is unknown. Physically, U_{dP} is the voltage of the converter under CP control mode. In a system, the number of related equations is limited, and only an equation set with small dimension should be solved. For small-scale problems, the efficiency of the Gauss–Seidel method is higher than Newton's method. Therefore, usually, the Gauss–Seidel method is employed to solve the rows related to U_{dP}. Assuming that $\boldsymbol{R} = \boldsymbol{G}^{-1}$, we have,

$$
\begin{aligned}
V_{dP} = {} & R_{PP}(U_{dP}^{-1} P_{dP}^{SP} - G_{PV} V_{dV}^{SP}) + R_{PV}(-G_{VV} V_{dV}^{SP}) + R_{PI}(I_{dI}^{SP} - G_{IV} V_{dV}^{SP}) \\
& + R_{PA}(DA_{dA}^{SP} - G_{AV} V_{dV}^{SP}) = R_{PP} U_{dP}^{-1} P_{dP}^{SP} + \Delta
\end{aligned}
\tag{10}
$$

where Δ can be calculated by the parameter of the HVDC system. Because the number of the converters under CP control modes is small, the Gauss–Seidel method is faster than Newton–Raphson method to solve $\boldsymbol{V}_{dP}$.

By submitting the right vector in (5) with the value of $\boldsymbol{V}_{dP}$, the $\boldsymbol{X}_{dc}$ can be solved by using (9).