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A multivariate dimension-reduction method for probabilistic power flow calculation

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ABSTRACT

The rising penetration of renewable generation as a result of environmental concerns generates increased uncertainties in power systems. This necessitates probabilistic analyses of the system performance, which include probabilistic power flow (PPF). The PPF suffers from the curse of dimensionality due to a large number of random loads. To address this issue, a multivariate dimension-reduction (MDR) method is proposed for PPF studies in this paper. The MDR decomposes the PPF problem into lower dimensional PPF subproblems which are further solved with promising accuracy. The computation time of the proposed method is proportional to the number of wind farms, which noticeably facilitates computation. The proposed method is applied to the IEEE 118-bus system and 2383-bus system. Simulation results demonstrate the accuracy and effectiveness of the proposed method.

1. Introduction

The increase in grid-connected large scale renewable energy generation brings about crucial operating challenges which stem from their random behavior. Reliable power flow analysis is an efficient interval analysis approach based on affine arithmetic [1,2]. It represents each uncertainty as an affine transformation of certain primitive variables and aims at the reliable estimation of the hull of power flow solution. This paper focuses on the probabilistic power flow (PPF). PPF plays an important role in studying the impact of uncertainties by considering the uncertainties in the power system as input random variables (such as wind generation and load demands) and calculating the statistical characteristics of the output variables (such as bus voltages and line flows). Performing a PPF study further assists system operators in making decisions for optimal operation and planning.

Various approaches have been proposed to address PPF problems with a tractable computational burden or favorable accuracy. These methods can be classified into three categories: Monte Carlo simulations (MCS) [3–5], analytical methods [6–8] and approximate methods [9–12]. MCS [3] has been widely used in PPF studies as the reference to validate the effectiveness of other PPF methods. Since MCS requires intensive repeated computation to acquire convergent and accurate results, Latin hypercube sampling (LHS) [4] and Latin supercube sampling (LSS) [5] techniques are proposed to improve the sampling efficiency of the MCS.

Analytical methods, such as the cumulant method (CM), are also extensively used because of their high computational efficiency. CM is used to obtain the moments of the output variables, which are then used to rebuild the distributions of output variables with asymptotic expansion theories [6,7], such as Gram–Charlier expansion, Edgeworth expansion and Cornish–Fisher expansion. The inclusion of wind farms in AC-PPF is proposed in Ref. [8]. However, it is a formidable task for the CM to deal with nonlinear correlated and non-Gaussian distributed renewable generation [7].

Approximation approaches are computationally efficient as well. These approaches involve repeated simulation with specified values of input variables to estimate the moments of output variables to rebuild the CDF of the output variables with asymptotic expansion theories. The point estimate method (PEM) is first introduced for the PPF evaluation in Ref. [9]. However, the PEM cannot estimate high order moments [10], and thus cannot provide the CDF of the output variables. The univariate dimension-reduction method (UDR) is capable of handling non-Gaussian random variables. UDR decomposes the high-dimensional function by a sum of univariate functions and has shown favorable potential in estimating high order moments [11,12]. The computation time of UDR

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Nomenclature

A. Definitions

PPF	Probabilistic power flow
CDF	Cumulative distribution function
PDF	Probability density function
MAE	Mean absolute error
RMSE	Root mean squared error

B. Constants

n_L	Number of active and reactive load demands
n_w	Number of wind farms
$\boldsymbol{X}_w^0$	Vector consisting of the median values of random wind power
$\boldsymbol{Z}_w^0$	Zero vector whose dimension is n_w
$\boldsymbol{X}_L^0$	Vector consisting of the median values of random load demands
$\boldsymbol{Z}_L^0$	Zero vector whose dimension is n_L
K	Number of quadrature points of the Gauss–Hermite formula
A_i	Weight coefficients of the Gauss–Hermite formula
N_{trials}	Number of trials of repeated deterministic load flow calculation required by PPF methods
C_{total}	Penetration level of wind power
$\boldsymbol{B}$	Inferior triangular matrix

C. Random variables and related parameters

$\boldsymbol{X_w}$	Vector of random wind power
$\boldsymbol{Z}_w$	Vector of independent standard normal random variables and $\boldsymbol{Z}_w = [z_w^1, \ldots, z_w^{n_w}]^T$
$\boldsymbol{X_L}$	Vector of random load demands
$\boldsymbol{Z}_L$	Vector of independent standard normal random variables and $\boldsymbol{Z}_L = [z_L^1, \ldots, z_L^{n_L}]^T$
$\boldsymbol{E}$	Vector of correlated standard normal random variables and $\boldsymbol{E} = [e_1, \ldots, e_{n_w}]^T$
F	Cumulative distribution function
F^{-1}	Inverse function of $F(\cdot)$
$\rho(e_i, e_j)$	Linear correlation coefficient between variables e_i and e_j
$\boldsymbol{C_E}$	Linear correlation matrix of $\boldsymbol{E}$
Φ	CDF of the standard normal random variable
$\rho_r(x_i, x_j)$	Rank correlation coefficient between variables x_i and x_j

D. Results of PPF

y	Output variables, including bus voltages and line flows
M_γ	γth central moments of $y(\boldsymbol{Z}_w, \boldsymbol{Z}_L)$
$M_{\gamma,L}$	γth central moments of $y(\boldsymbol{Z}_w^0, \boldsymbol{Z}_L)$
$M_{\gamma,i}$	γ^{th} central moments of $y(\boldsymbol{Z}_w^i, \boldsymbol{Z}_L^0)$

E. Error indices

ε	Average of MAE with respect to the output variables within the same category
ε_r	Average of RMSE with respect to the output variables within the same category

is proportional to the number of all random variables. The high dimensionality of the PPF problem due to a large number of random loads may slow down the UDR and make it less attractive in large-scale problems.

This paper proposes a multivariate dimension-reduction (MDR) method for PPF calculation. The output variable in the PPF problem is regarded as a function of input variables. The MDR approximates such high dimensional function by a sum of lower dimensional functions and decomposes the PPF problem into two PPF subproblems. The subproblems are further solved with high accuracy and computational efficiency using the linearization technique and Gauss-Hermite quadrature rules, respectively. MDR has following contributions:

1) MDR aims at the practical situation that the power system has lots of random loads. Inspired by the UDR, MDR is derived from multivariate Taylor series according to the characteristics of PPF and utilizes the advantages of two methods. The linearization technique can deal with lots of random loads while the Gauss-Hermite quadrature rules are suitable for nonlinear correlated wind power. High accuracy can be achieved. To the best of our knowledge, few literatures make such efforts.
2) The number of repeated trails of power flow calculation MDR requires is just proportional to the number of wind farms. The MDR realizes substantial improvement over the UDR in terms of the run time.

The rest of the paper is organized as follows. Section 2 gives an introduction to PPF problems. Section 3 illustrates the MDR and describes the procedure of applying MDR to the PPF problem. The performance of MDR is investigated with the modified IEEE 118-bus system and 2383-bus system in Section 4. Section 5 concludes the paper.

2. PPF problem

2.1. Power flow formulation

The deterministic power flow equations for the power system can be written as:

$$0 = P_i^{inj} - V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\theta_i - \theta_j - \varphi_{ij}) \tag{1}$$

$$0 = Q_i^{inj} - V_i \sum_{j=1}^{N} V_j Y_{ij} \sin(\theta_i - \theta_j - \varphi_{ij}) \tag{2}$$

where P^{inj} and Q^{inj} are real and reactive power injections, and V and θ are the nodal voltages and angles; $Y_{ij}\angle\varphi_{ij}$ is the (i,j)th element of the admittance matrix; and N is the number of nodes in the power network.

The input variables can be categorized into two groups: loads and renewable generation (wind generation in this paper). The state variables of the power system, such as line flows, are regarded as the output variables and are given as functions of the input variables, which are determined by Eqs. (1) and (2).

2.2. PPF formulation

Compared to the deterministic power flow, the PPF analyses the statistics of output variables resulted from the random nature of input variables. Since it is more convenient to solve PPF problems involving independent standard normal variables than actual random variables, the proposed method is illustrated using independent standard normal variables. These independent standard normal variables can be transformed into the actual nonlinearly correlated and non-Gaussian random variables using normal copula [13,14]. The Nataf transformation is inherently an equivalent of the normal copula [11]. Then, the proposed method is also applicable to non-Gaussian random variables.