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Quasi-static analysis of lightning-radiated electric field in the vicinity of sea-land structure

Ehsan Zareian-Jahromi*, Reza Sarraf Shirazi, Mojtaba Khosravi-Farsani, Seyed Hossein H. Sadeghi, Rouzbeh Moini

Electromagnetics Research Laboratory, Amirkabir University of Technology, 424 Hafez Avenue, Tehran 15914, Iran

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ABSTRACT

A quasi-static approach is presented to compute electric field radiated by a lightning return stroke above the sea-land structure modeled as a dielectric half space with a perfectly conducting half plane situated in its interface. It appears that the quasi-static approach provides a closed-form time-domain solution for the radiated electric field. The proposed approach is mainly based on the electrostatic solution of the structure. The governing Poisson's equation in the presence of the sea-land structure is solved in three dimensions, using the contributions of dielectric half space and perfectly conducting half plane. The solution, expressed in terms of image contributions, consists of two point images in physical space. The validity domain of the time-domain quasi-static approach is investigated by comparison of the obtained electric fields with full-wave results obtained using the finite-difference time-domain method. The proposed approach has a reasonable accuracy at near (200 m) distance range from the lightning channel.

1. Introduction

Many electric power installations such as wind turbines, wave energy converters, and transmission lines are located near shorelines [1–3]. Electric field radiated by a lightning return stroke can cause malfunction or destruction of the power installations. The analysis of the lightning electric field is necessary to efficiently design a lightning protection procedure. This analysis is highly dependent upon the propagation path which may be a homogenous or stratified medium. The stratified medium can be classified into horizontally or vertically stratified medium. The analysis of lightning electric field has been widely studied in literature [4–8].

For the case of a horizontally stratified ground, Shoory et al. [9] studied the simplified analytical expressions derived by Wait [10] at far distances of 10–100 km from the lightning channel. The validity of simplified approaches has been studied in [11] for the vertical electric field above the interface of a horizontally stratified medium, and found that results using the simplified approaches are in excellent agreement with full-wave results in very close (50 m) and intermediate (1 km) distance ranges.

The propagation effect of a sea-land path considered as a vertically stratified ground on the lightning electric field has been analyzed in [12,13] using Wait's formula [14,15]. The accuracy of Wait's formula has been examined in [16,17] by comparison with full-wave results obtained using the finite-difference time-domain (FDTD) method. The good accuracy of Wait's simplified formulas has been shown for far distances from the lightning channel (10–100 km) in [16] and near (200 m) and intermediate (1 km) distance ranges from the lightning channel in [17].

The Wait's formula should be calculated for many frequency samples over the frequency range of the lightning channel current to achieve the time-domain radiated electric field. Therefore, we are more interested in a closed-form time-domain solution of the contribution of sea-land path on the lightning electric field. In this paper, a novel quasi-static time-domain approach is proposed based on solving the Poisson's equation in the presence of sea-land structure. The sea-land structure is modeled as a dielectric half space with a perfect electric conductor (PEC) half plane situated in its interface.

The paper is organized as follows: In Section 2, the simplified model of the sea-land structure is introduced. An electrostatic image theory for solving the Poisson's equation in the presence of this geometry is developed. Based on these electrostatic images, a novel time-domain quasi-static approach is proposed to analyze the radiated electric field by an electric current dipole in the vicinity of the structure. In Section 3, the closed-form time-domain formulation of lightning electric field over the proposed structure is

* Corresponding author. Tel.: +98 21 64543397.
E-mail addresses: e.zareian@aut.ac.ir, ehsan_zareian@yahoo.com (E. Zareian-Jahromi).

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reviewed and discussed. The validity domain of the obtained formulations is also tested *versus* full-wave simulations obtained using FDTD. Conclusions are given in Section 4.

2. Theory

2.1. Electrostatic solution of the problem

The sea-land structure can be assumed as a PEC half plane situated in the interface of free space and a dielectric half space medium [18]. The three-dimensional Poisson's equation is considered in the presence of the proposed structure. The geometry of the problem is illustrated in Fig. 1. It should be mentioned that sea water fulfills the assumption of perfect electric conductivity to an adequate degree of accuracy [18].

The dielectric constants outside and inside of the dielectric half space are ε_1 and ε_2, respectively. The electrostatic potential in the exterior region, ϕ_1, satisfies the Poisson's equation:

$$\nabla^2\phi_1(\rho,\varphi,z) = -\frac{1}{\varepsilon_1\rho}\delta(\rho-\rho_0)\delta(\varphi-\varphi_0)\delta(z-z_0) \quad (0\le\varphi\le\pi), \tag{1}$$

where (ρ_0, φ_0, z_0) reveals the position of the unit point charge in cylindrical coordinates. The electrostatic potential in the interior region, ϕ_2, satisfies the Laplace's equation:

$$\nabla^2\phi_2(\rho,\varphi,z) = 0 \quad (\pi\le\varphi\le 2\pi). \tag{2}$$

First, we consider the electrostatic contribution of the PEC half plane in free space. The electrostatic potential of a unit point charge in the vicinity of the PEC half plane in free space filled with ε_n, ψ_n, has been presented in a closed-form representation in [19]. The solution contains some typographical errors which can be corrected by substituting $(2\pi-\varphi_0)$ with $-\varphi_0$ and considering $0\le\tan^{-1}x\le\pi$ in Eqs. (63)–(67) of [19]. The correct representation of ψ_n can be obtained from:

$$\psi_n(\rho,\varphi,z,\varphi_0) = \frac{1}{4\pi\varepsilon_n}\left[\frac{1}{R(\varphi_0)} - \frac{1}{R(-\varphi_0)} - \frac{\tan^{-1}\left(\frac{R(\varphi_0)}{g(\varphi_0)}\right)}{\pi R(\varphi_0)} + \frac{\tan^{-1}\left(\frac{R(-\varphi_0)}{g(-\varphi_0)}\right)}{\pi R(-\varphi_0)}\right], \tag{3}$$

where $R(\varsigma)$ is the distance between points (ρ, φ, z) and (ρ_0, ς, z_0) written in cylindrical coordinates:

$$R(\varsigma) = \sqrt{\rho^2+\rho_0^2+(z-z_0)^2-2\rho\rho_0\cos(\varphi-\varsigma)}, \tag{4}$$

and

$$g(\varsigma) = 2\sqrt{\rho\rho_0}\cos\left(\frac{\varphi-\varsigma}{2}\right). \tag{5}$$

It can be easily seen that the electrostatic potential distribution meets the proper singularity and boundary conditions according to the contributions of the unit excitation point charge, the

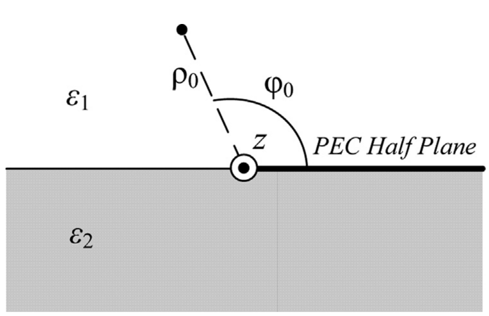

Fig. 1. Point source in the vicinity of the model of the sea-land structure.

−1 image point charge, and two different image charges with variable amplitudes according to the observation point. The solution of the electrostatic potential distribution in the presence of the structure depicted in Fig. 1 can be obtained using Eq. (3) and the electrostatic image theory of the point source in the presence of a dielectric half space filled with ε_2. The electrostatic potential in the exterior and interior regions can be defined as:

$$\phi_1(\rho,\varphi,z) = \psi_1(\rho,\varphi,z,\varphi_0) - \Gamma\psi_1(\rho,\varphi,z,2\pi-\varphi_0), \tag{6}$$

$$\phi_2(\rho,\varphi,z) = T\psi_2(\rho,\varphi,z,\varphi_0), \tag{7}$$

where Γ and T are defined as:

$$\Gamma \triangleq \frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}, \tag{8}$$

$$T \triangleq \frac{2\varepsilon_2}{\varepsilon_2+\varepsilon_1}. \tag{9}$$

It can be easily seen that the proposed solutions for ϕ_1 and ϕ_2 meet the proper singularity and boundary conditions. After some analytical simplifications, ϕ_1 can be obtained from:

$$\phi_1(\rho,\varphi,z) = \frac{1}{4\pi\varepsilon_1}\left[\frac{\alpha_{11}}{R(\varphi_0)} + \frac{\alpha_{12}}{R(-\varphi_0)}\right], \tag{10}$$

where

$$\alpha_{11} = 1 - \frac{(1-\Gamma)}{\pi}\tan^{-1}\left(\frac{R(\varphi_0)}{g(\varphi_0)}\right), \tag{11}$$

$$\alpha_{12} = -1 + \frac{(1-\Gamma)}{\pi}\tan^{-1}\left(\frac{R(-\varphi_0)}{g(-\varphi_0)}\right). \tag{12}$$

Therefore, ϕ_1 can be expressed as the contributions from the excitation point source multiplied by α_{11} and its mirror image multiplied by α_{12} situated in free space filled with ε_1. For the case of the interior region, ϕ_2 can be expressed as follows:

$$\phi_2(\rho,\varphi,z) = \frac{1}{4\pi\varepsilon_2}\left[\frac{\alpha_{21}}{R(\varphi_0)} + \frac{\alpha_{22}}{R(-\varphi_0)}\right], \tag{13}$$

where

$$\alpha_{21} = T\left[1 - \frac{1}{\pi}\tan^{-1}\left(\frac{R(\varphi_0)}{g(\varphi_0)}\right)\right], \tag{14}$$

$$\alpha_{22} = T\left[-1 + \frac{1}{\pi}\tan^{-1}\left(\frac{R(-\varphi_0)}{g(-\varphi_0)}\right)\right]. \tag{15}$$

It is worth noting that ϕ_2 can be expressed as the contributions from the excitation point source multiplied by α_{21} and its mirror image multiplied by α_{22} situated in free space filled with ε_2.

2.2. Electric current dipole over the sea-land structure

To solve the electric field radiation of a vertical electric current dipole in the presence of the structure depicted in Fig. 1, it is assumed that a vertical current source $i(t)\boldsymbol{n}$ is located in the exterior region of the proposed structure (see Fig. 2). It should be mentioned that $\boldsymbol{n}$ denotes a unit vector normal to the PEC half plane. In the quasi-static regime, it is assumed that the electric current dipole can be replaced by proper electric dipole containing two opposing point charges q and $-q$ at the top end and bottom end of the electric current dipole, respectively (see Fig. 3). First, the electrostatic analysis of the excitation electric dipole in Fig. 3 is obtained using Eqs. (10)–(15). Afterwards, q is replaced by [20]:

$$q = \int_{t_0}^{t} i(t')\ dt', \tag{16}$$