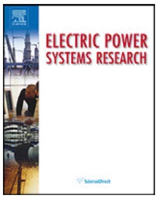
Electric Power Systems Research 84 (2012) 45–57

Contents lists available at SciVerse ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr



Linear power flow *V*-theta

Ailson P. de Moura[a,*], Adriano A.F. de Moura[b], D.S. Oliveira Jr.[a], E.J. Fernandes[a]

[a] *Department of Electrical Engineering, Federal University of Ceara Technology Center, Pici Fortaleza-CE, CEP: 59.810-321, Brazil*
[b] *Department of Environmental Science and Technology, Federal Rural University of Semi-Arid, Av. Francisco Mota, 572 Bairro Costa e Silva, Mossoró-RN, CEP: 59.625-900, Brazil*

ARTICLE INFO

Article history:
Received 26 May 2011
Received in revised form
23 September 2011
Accepted 27 September 2011
Available online 28 October 2011

Keywords:
Linear model
DC power flow
Power flow
Contingency analysis

ABSTRACT

This paper presents a new linear active power flow solution method. The new linear active power flow presents a better performance to calculate MW power flows, as opposed to MW flows, calculated in classical model with a DC power flow. The aforesaid proposed method is based on a decoupling principle. Therefore, the voltage angles and voltage magnitudes are calculate in decoupled forms. A complete demonstration of the proposed method is presented. The algorithm of new method has been tested by extensive numerical studies. This paper gives details of the method's performance on various classical systems. All the results are compared with those from the Newton–Raphson power flow and classical DC power flow.

1. Introduction

The power flow calculation is one of the most commonly used tools in power system engineering. For that reason, the history of power flow calculation is a relatively long one. Since the invention and widespread of computers, in the 1950s and 1960s, many methods for solving the power flow problem have been developed [1].

The classical direct current (DC) power flow technique calculates only real power flows within power systems networks. The DC power flow method was especially attractive in the middle of the twentieth century, when computer access was expensive, and there was a real need to reduce central processing unit (CPU) time on all computational activities. Presently, the DC power flow method is used extensively in power system analysis and power market applications [2,3]. Several examples are presented herewith: (A) Contingency Analysis – Contingency analysis is the focal point in evaluating power system security. The DC power flow method is preferred in this analysis [4]; (B) Calculation of Power Transfer Distribution Factors (PTDF) – The PTDF represents the sensitivities of lines flow with respect to generation changes [4]. The PTDFs are used in transmission congestion management applications and also in the calculation of Locational Marginal Price (LMP) [5,6]; (C) Calculation of Line Outage Distribution Factors (LODF) – The LODF measures the sensitivity of a line flow against the removal of another line. The LODF is used in the calculation of available transfer capability (ATC), as well as developing constraints for SPD (Scheduling, Pricing and Dispatch) and Security Constrained Unit Commitment (SCUC) programs [5,7,8]; (D) Transmission Interchange Limit Analysis – The Transfer Limit Table Generator (TLTG) and POLY analyses are transmission interchange limit analysis functions in Power System Simulator for Engineering (PSS/E) [7], which estimate the import/export limits between two areas ("study system" and "opposing system") using linearized network model. The POLY function differs from the TLTG in that it considers simultaneous generation shifts in two opposing systems to maximize study system import/export; (E) Market Clearing Engine (MCE) – The Scheduling, Pricing, and Dispatch (SPD), Security Constrained Unit Commitment (SCUC) and Simultaneous Feasibility Test/Network Application (SFT/NA) are core programs of an MCE. The SPD and SCUC are security constrained optimal power flow programs, often based on the linear DC power flow equations. The objective is to minimize the total cost of generation and reserves, subject to a set of constraints including power balance, ancillary services, resources operating limits and transmission security constraints; (F) Financial Transmission Right – Financial transmission right (FTR), also known as firm transmission right, is a financial instrument for hedging risks from transmission congestion costs on constrained lines [9]. A linear programming (LP) problem is formulated to clear the FTR auction. The objective function of the LP problem is to maximize the revenues from FTR. The thermal limits of the transmission lines are formulated as power flow constraints of

---

[*] Corresponding author. Tel.: +55 85 32413528.
E-mail address: ailson@ufc.br (A.P. de Moura).

0378-7796/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.epsr.2011.09.020

the LP problem. Others developments in this area are comparisons of MW flows obtained from DC and AC power flow solutions [10], and methods that try to account for the Mvar flows that are absent from DC models [11].

The advantages of a DC model are as follows:

(a) Its solutions are non-iterative, reliable and unique;
(b) Its methods and software are relatively simple;
(c) Its models can be solved and optimized efficiently, particularly in the demanding area of contingency analysis;
(d) Its network data is minimal and relatively easy to obtain;
(e) Its linearity fits the economic theory on which much of transmission-oriented market design is based;
(f) Its approximated MW flows are reasonably accurate, at least for the heavily loaded branches that might constrain system operation.

These are powerful attractions and, general, items (a)–(e) are mostly valid. However, it is well known that the DC power flow method offers only approximate solutions, especially when the R/X ratios for transmission lines are large and bus voltages are highly non-uniform. This inaccuracy leads to compromised system reliability when used in system security analysis, and can have economic consequences by changing the LMP in security constrained economic dispatch or FTR awards in FTR auctions [8].

The method described in this paper presents a better performance to calculate MW power flows than MW flows calculates in classical model of a DC power flow. Great importance is presented to the fact that this proposed method, is derived from some theoretical bases. The method is based on decoupling principles, and it uses a procedure that solves the voltage angles and voltage magnitudes in decoupled forms. The new method can calculate active and reactive power flows. However, due to page limit, the reactive power flows will be presented and validated in future paper.

The paper is organized as follows: Firstly, a complete demonstration of the proposed method is presented, inclusive with numerical examples; all the necessary approximations and details required by proposed method are identified. Test results are presented with some relevant conclusions duly reported.

2. Development of the linear power flow *V*-theta

Consider expressions for the active power flows in a transmission line. These expressions are explained briefly in Appendix A, where a summary of the DC power flow is presented. The following approximations are made:

$$V_k^2 \cong factorV_k \tag{1}$$

$$V_m^2 \cong factorV_m \tag{2}$$

$$V_kV_m \cong factorV1_{km} \tag{3}$$

$$\sin \quad \theta_{km} \cong \theta_{km} \tag{4}$$

$$\cos \quad \theta_{km} \cong 1 \tag{5}$$

Then, the linearized expressions for the active power flow in a transmission line are as follows:

$$P_{km}^{LIN} = factorV_k g_{km} - factorV1_{km} g_{km} - factorV1_{km} b_{km} \theta_{km} \tag{6}$$

$$P_{mk}^{LIN} = factorV_m g_{km} - factorV1_{mk} g_{km} + factorV1_{mk} b_{km} \theta_{km} \tag{7}$$

where the superscript “LIN” means linearized.

g_{km} and b_{km} are respectively series conductance and series susceptance.

$$factorV_k = cV_k + d$$

$$factorV_m = eV_m + g$$

c, d, e, and g are constants, i.e. the $factorV_k$ and $factorV_m$ are approximated as linear functions; where V_k and V_m are numerical values of voltage magnitudes read from the data file.

$factorV1_{km}$ is the linear approximation of the tangent plane to a surface at a point stays close to the surface near the point [12]. At point (a, b) $factorV1_{km} = f(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$.

The $factorV_k$ and $factorV_m$ are calculated at intervals of 0.005 p.u. The $factorV1_{km}$ is calculated at point $a = V_k + k$ and $b = V_m + k$ both in p.u., where k is a constant. Therefore, the local linearization of $f(x, y) = xy$ at point (a, b) is

$$factorV1_{km} = (V_m + k)x + (V_k + k)y - V_kV_m - kV_k - kV_m - k^2 \tag{8}$$

When $x = V_k$ and $y = V_m$ in Eq. (8), $factorV1_{km}$ is

$$factorV1_{km} = V_kV_m - k^2 \tag{9}$$

The calculation of the voltage angles and voltage magnitudes, in a linearized form, are made as follow: Two equations of active power mismatches (ΔP) and two equations of power reactive mismatches (ΔQ), are written in matrix form and they are linearized. The first two equations have ΔP and ΔQ with the voltage angles as unknowns. The next two equations present ΔP and ΔQ with voltage magnitudes as unknowns. The four equations form an overdetermined system of linear equations, which is solved by a mathematical procedure already known [13,14].

The others assumptions made within the linear power flow V-theta technique are as follows: