Applied Thermal Engineering 106 (2016) 731–742

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Research Paper

Estimation of spatially varying thermal contact resistance from finite element solutions of boundary inverse heat conduction problems split along material interface

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HIGHLIGHTS

- A simple method for estimating thermal contact resistance is presented.
- Thermal contact resistance as constant, sinusoidal and triangle functions are tested.
- Quantifying the uncertainty of estimated thermal contact resistance is demonstrated.

ARTICLE INFO

Article history:
Received 19 March 2016
Revised 31 May 2016
Accepted 9 June 2016
Available online 11 June 2016

Keywords:
Inverse heat conduction problem (IHCP)
Thermal contact resistance
Finite element method
Modified cubic spline

ABSTRACT

This paper presents a method for estimating spatially varying thermal contact resistance from a computational point of view. The method starts by splitting the computational domain along material interface to yield two boundary inverse heat conduction problems. Temperatures computed from analytical solutions are specified at only three interior points of each material instead of temperatures available from experiments. A number of equations constructed from modified cubic spline specified along the interface are incorporated into finite element equations during the solution process. After extracting temperatures and heat transfer rates from both finite element solutions, which are solved separately, thermal contact resistance at each pair of coincident nodes on the interface are calculated.

Constant, sinusoidal, and triangle functions are selected as spatial functions of thermal contact resistance along material interface to test the method. Constant thermal contact resistance is estimated accurately. Shapes of sinusoidal and triangle functions are fairly captured, locations of their maximums are correctly predicted. Thermal load is suggested to be applied on the boundary of material that has lower thermal conductivity to let the method works properly. Quantifying the uncertainty of estimated thermal contact resistance is demonstrated by adding bias error based on accuracy of sensors to the temperatures specified on the interior points of both materials.

1. Introduction

Thermal contact resistance is an important parameter in heat conduction of materials that are in contact because it limits rate of heat removal from composite regions in many applications, such as electronic packing [1], nuclear reactors [2], aerospace, and biomedicine [3]. Estimation of thermal contact resistance is very difficult in practice because it depends on several parameters including type of materials in contact, hardness and roughness of material surfaces, contact pressure distribution [4], and temperature distributions in materials [5].

Although theoretical predictions of thermal contact resistance are available, for example Mikic and Rohsenow [6] and Cooper et al. [7], they should be corroborated with experimental data. Furthermore, researches on thermal contact resistance estimation reported it as a global value, for example Halliday et al. [8], Wolff and Schneider [9], Zhang et al. [1], Yang [10], Fieberg and Kneer [11], Liu et al. [12], and Luo et al. [13]. Recently, estimations of spatially varying thermal contact resistance were investigated by Gill et al. [4], Colaço and Alves [14], and Colaço et al. [15]. Although their results are accurate, difficulties arise during intrusive collecting of temperatures and non-intrusive collecting of both temperatures and heat transfer rates from many locations in experiments. Consequently, a method for accurate estimation of spatially

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http://dx.doi.org/10.1016/j.applthermaleng.2016.06.060
1359-4311/

Nomenclature

a, b	side lengths of rectangular element in x-, y-direction (m)
$\mathbf{b}$	modified vector of thermal loads (–)
f	function for temperature distribution along a line (K)
k	thermal conductivity (W/(m · K))
q	heat flux (W/m^2)
$\mathbf{r}$	vector of residuals (–)
s	coordinate along domain boundary (m)
t	element thickness (m)
x, y	global coordinates (m)
$\mathbf{x}$	vector of unknowns (–)
A	area of two-dimensional domain (m^2)
$\mathbf{A}$	modified coefficient matrix (–)
F	factor obtained from triple integration (–)
$\mathbf{I}$	identity matrix (–)
$\mathbf{K}$	coefficient matrix (W/K)
N	element interpolation function (–)
$\dot{Q}$	heat transfer rate (W)
R	thermal contact resistance (m^2 · K/W)
S	planar heat source strength (W/m^2)
T	temperature (K)
$\times$	non-zero matrix (–)

Greek symbols

δ	vertical distance from horizontal edge of the domain (m)
ε	relative error (–)
η	thickness of upper material (m)
ξ, η	local coordinates (m)

Subscripts

1, 2	material number
in	entering the domain
out	leaving the domain
x, y	component in x-, y-direction

Superscripts

0	initial guess
e	system of equations of element
k	iteration number
s	system of equations of the domain

varying thermal contact resistance that requires fewer data collecting locations is needed.

Generally, function specification and regularization methods [16–19] can be employed for estimating realistic values of thermal contact resistance without any limitation on number of sensors used in experiments. Although the number of sensors cannot be chosen arbitrarily in this paper, the method presented can estimate spatially varying thermal contact resistance by using a small number of sensors. Optimization based on the least squares method that solves inverse heat conduction problems is not analyzed here. Instead, a finite element formulation used in solving direct heat conduction problem is modified to predict temperatures and heat transfer rates on material interfaces when temperatures at some locations within each material are available. Then, thermal contact resistance at each pair of coincident nodes on the interface can be estimated.

The objective of this paper is to present a simple method that requires less temperature data from experiments for estimating spatially varying thermal contact resistance at steady state and to demonstrate quantification of uncertainty of estimated thermal contact resistance. All temperatures used as input for all problems are synthesized from analytical solutions to initiate the idea in applying the method in thermal experiments. The concept presented in this paper begins by splitting the computational domain along material interface to yield two boundary inverse heat conduction problems. Constant, sinusoidal, and triangle functions are selected as spatial functions for thermal contact resistance along the interface to test the method. Analytical solutions resulting from these functions are derived in Section 2 to provide temperatures at only three interior points of each material to simulate measuring of temperatures from experiments. A finite element formulation incorporated with modified cubic spline specified along a portion of the domain boundary is developed in Sections 3.1 and 3.2 to predict temperatures and heat transfer rates at the interface of each material. Calculation of thermal contact resistance at each pair of coincident nodes along the interface is proposed in Section 3.3. Effects of mesh size and locations of specified temperatures provided from analytical solutions are briefly studied in Section 4.1 to select a combination of these parameters that will be employed in remaining sections. The effect of thermal conductivity ratio is examined in Section 4.2 and the quantifying of uncertainty of estimated thermal contact resistance is demonstrated in Section 4.3.

2. Analytical solutions of steady-state heat conduction problems

In this section, analytical solutions of steady-state heat conduction in a rectangular domain occupied by two rectangular shaped materials are derived for constructing of problems on estimating thermal contact resistance along the interface found in Section 4.

2.1. General case

Assuming that both materials are isotropic and homogeneous with constant properties and there is no volumetric heat generation within them, governing equations for the steady-state heat conduction problem shown in Fig. 1 are

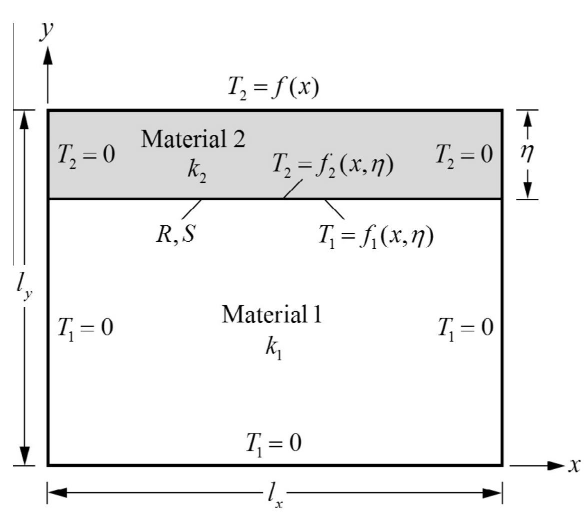

Fig. 1. Steady-state heat conduction in rectangular domain occupied by two rectangular shaped materials.