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Research Paper

A nonlinear fin design problem in determining the optimum shapes of fully wet annular fins

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HIGHLIGHTS

- The nonlinear fully wet annular fins is designed based on the desired fin efficiency.
- The temperature-dependent thermal properties of annular fins are considered.
- The numerical results are examined to justify the validity of the fin design problem.
- The Biot numbers for ambient air can affect significantly the optimal fin shape.
- The designed optimal fin shape helps to improve the efficiency of annular fin.

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ABSTRACT

Because the thermal conductivity of an annular fin is a function of temperature in many practical engineering applications, the nonlinear optimum shape design problem is considered in this study. The conjugate gradient method (CGM) is utilized as the optimization tool based on the desired fin efficiency and fin volume. It is assumed that the surrounding air has 100% relative humidity, and thus an annular fin under fully wet conditions can be assured. The numerical experiment results show that the optimum annular fin has the highest fin efficiency among six annular fins with the same operating fin conditions. When the Biot numbers for ambient air (Bi_a) varied, the optimum fin efficiency and optimum fin shape of the nonlinear fully wet annular fin also changed significantly. However, when the nonlinearity of the Biot numbers for the inner tube (Bi_i), the thermal conductivities of the bare tube (k_w) and the annular fin (k_f) varied, the optimum fin shape remained almost the same. This implies that those three thermal parameters have a limited influence on the optimum fin shape for an annular fin and the results obtained from this work could be utilized directly to the evaporator manufacturing industries.

1. Introduction

The temperatures of the surface of an annular fin adhered to a bare tube on the evaporator is generally below the dew point of the surrounding air. Heat and mass transfer occur simultaneously, and moisture condenses on the fin surface resulting in a fully wet condition. Because the thermal conductivities of annular fins and bare tubes are a function of temperature in many practical engineering applications, the nonlinear fin shape design problem becomes important. For this reason a rectangular nonlinear fully wet annular fin adhered to a bare tube is re-designed in this study to obtain the optimum fin shape and to improve the refrigeration effect.

Many researchers have reported heat and mass transfer analyses for a linear wet annular fin to determine the temperature distributions on the fin surface and the fin efficiency. For example, Sharqawy and Zubair [1] considered heat and mass transfer mechanisms simultaneously and derived closed-form solutions for the efficiency of fully wet annular fins with different configurations. McQuiston [2] determined the one-dimensional fin efficiency of a fully wet rectangular longitudinal fin. Naphon [3] examined the heat transfer characteristics of an annular fin under dry, partially wet, and fully wet surface conditions and presented the theoretical results of the heat transfer characteristics and the fin efficiency of the annular fin.

Toner et al. [4] used a quasi-linear one-dimensional model to analyze rectangular and triangular fins for when moisture condensation occurs on the fin surfaces. Wu and Bong [5] were the first to derive an analytical solution for the efficiency of a longitudinal straight fin under dry, fully wet, and partially wet surface

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Nomenclature

$\bar{A}(\bar{r})$	cross-sectional area of the fully wet annular fin (m^2)
$Bi_a(\theta_f)$, $Bi_i(\theta_w)$	temperature-dependent Biot number
$\bar{h}_o(\bar{T}_f), \bar{h}_b(\bar{T}_w)$	dimensional convective heat transfer coefficient
$\bar{k}_w(\bar{T}_w)$, $\bar{k}_f(\bar{T}_f)$	dimensional temperature-dependent thermal conductivity
$k_w(\theta_w)$, $k_f(\theta_f)$	non-dimensional temperature-dependent thermal conductivity
$\bar{h}_d$	mass transfer coefficient (kg m^{-2} s^{-1})
$\bar{h}_{fg}$	latent heat of condensation of moisture (J kg^{-1})
$C_{p,a}$	constant pressure specific heat for moist air (J kg^{-1} K^{-1})
ϕ	relative humidity
J	functional defined by Eq. (3)
J'	gradient of the functional defined by Eq. (15)
L_e	Lewis number
$\bar{P}$	half fin pitch (m)
q	actual heat transfer rate through the fully wet annular fin
Q	ideal heat transfer rate through the fully wet annular fin
$\bar{r}_b$	inner radius of the tube (m)
$\bar{r}_o$	external radius of the tube (m)
$\bar{r}_t$	external radius of the fin (m)
$\bar{S}(\bar{r})$	perimeter of the fully wet annular fin (m)
$\bar{T}$	temperature (°C)
$v(\delta)$	estimated annular fin volume
V	specified annular fin volume

Greeks

α	weighting coefficient
β	search step size
γ	conjugate coefficient
$\delta(r)$	thickness of the fully wet annular fin
$\theta_w(r)$, $\theta_f(r)$	estimated dimensionless wall and fin temperature
$\Delta\theta_w(r)$, $\Delta\theta_f(r)$	sensitivity function of wall and fin defined by Eq. (7)
$\lambda(r)$	Lagrange multiplier defined by Eq. (13)
$\Gamma(\bullet)$	Dirac delta function
η	fully wet annular fin efficiency
Φ	desired fin efficiency
ϕ	relative humidity
ε	convergence criterion
$\bar{\omega}_f$	specific humidity of air on the annular fin surface
$\bar{\omega}_h$	specific humidity of the surrounding air

Superscript

n	iteration index
_	dimensional quantities

conditions with consideration of the temperature and humidity ratio differences as the driving forces for the heat and mass transfer processes. Kundu [6] analytically determined the performance of straight tapered longitudinal fins that were subject to simultaneous heat and mass transfer.

Recently, Al-Jewaree and Alhami [7] investigated experimentally the performance of annular fins of different shapes subject to locally variable heat transfer coefficient. Experimental results indicates that for the considered conditions the performance of heat transfer rate to elliptical fin is better than those for diamond and circular fins. Moreover, due to the practical importance, the influence of the temperature-dependent thermal properties on the temperature distributions of annular fin was examined by Aksoy [8], Moradi and Ahmadikia [9] and Ganji et al. [10].

Numerous studies have been conducted to optimize the dimensions of linear fully wet annular fins that are subjected to convection. For instance, the optimum dimensions of uniform fully wet annular fins were studied by Brown [11], and the optimum performance and fin length of fully wet annular fins with a rectangular profile were presented by Kang [8] using a variation separation method. Kang [12] concluded that when considering the thermal conductivity to be constant, the optimum base thickness and the volume of the fin are inversely proportional to the thermal conductivity of the fin material; whereas, the optimum length and effectiveness are independent of the properties of the material used. Recently, Huang and Chung [13] applied the conjugate gradient method to a fully wet annular fin design problem to design optimum shapes based on the desired fin efficiency and fin volume.

In all of the references above, the optimum shape of the fins was determined based on constant thermal properties; fin design problems based on the temperature-dependent thermal properties of a fin are very limited in the literature.

For this reason, Huang and Hsiao [14] used the conjugate gradient method (CGM) [15] in a nonlinear fin design algorithm to estimate the optimum shapes for the spine and longitudinal fins based on the desired fin efficiency and fin volume. Huang and Chung [16] considered a nonlinear fully wet fin design problem in determining the optimum shapes for the longitudinal and spine fins also using the CGM based on the desired fin efficiency and fin volume.

The objective of the present work is to develop a fin design algorithm using the CGM to estimate the optimum shapes of fully wet annular fins adhered to a bare tube based on the desired fin efficiency when the thermal properties are a function of temperature. This approach could match with the operational condition of evaporator in reality.

2. The direct problem

The relative humidity of the external environment, ϕ, is assumed to be 100% in the present study, and therefore the temperature of the surrounding air becomes the dew point temperature.

The dimensional governing equations and the boundary conditions for a steady-state fully wet nonlinear annular fin adhered to a bare tube are given as follows [3]:

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left[\bar{r}\bar{k}_w(\bar{T}_w)\frac{d\bar{T}_w(\bar{r})}{d\bar{r}}\right]=0;\ in\ \bar{r}_b\leqslant\bar{r}\leqslant\bar{r}_o \tag{1a}$$

$$\frac{d}{d\bar{r}}\left[\bar{k}_f(\bar{T}_f)\bar{A}(\bar{r})\frac{d\bar{T}_f(\bar{r})}{d\bar{r}}\right]=\bar{h}_o(\bar{T}_f)\bar{S}(\bar{r})\left\{[\bar{T}_f(\bar{r})-\bar{T}_h]+\frac{\bar{h}_{fg}\bar{h}_d}{\bar{h}_o(\bar{T}_f)}[\bar{\omega}_f(\bar{T}_f)-\bar{\omega}_h]\right\};\ in\ \bar{r}_o\leqslant\bar{r}\leqslant\bar{r}_t \tag{1b}$$

$$-\bar{k}_w(\bar{T}_w)\frac{d\bar{T}_w}{d\bar{r}}=\bar{h}_b(\bar{T}_w)(\bar{T}_c-\bar{T}_w);\quad \text{at } \bar{r}=\bar{r}_b \tag{1c}$$

$$\bar{T}_w=\bar{T}_f;\quad \text{at } \bar{r}=\bar{r}_o \tag{1d}$$

$$-\bar{k}_w(\bar{T}_w)\bar{P}\frac{d\bar{T}_w}{d\bar{r}}=\bar{h}_o(\bar{T}_f)\left(\bar{P}-\frac{1}{2}\bar{\delta}\right)\left\{(\bar{T}_w-\bar{T}_h)+\frac{\bar{h}_{fg}\bar{h}_d}{\bar{h}_o(\bar{T}_f)}[\bar{\omega}_f(\bar{T}_f)-\bar{\omega}_h]\right\}-\frac{1}{2}\bar{k}_f(\bar{T}_f)\bar{\delta}\frac{d\bar{T}_f}{d\bar{r}};\ \ \text{at } \bar{r}=\bar{r}_o \tag{1e}$$