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Research Paper

Modeling energy conversion in a tortuous stack for thermoacostic applications

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ABSTRACT

The stack represents the core of standing wave engines since inside it the thermal energy is converted into mechanical energy. Commonly stacks are realized with straight pores whose sections have regular shapes (e.g. circular, rectangular). In these cases the viscous and thermal interactions are described by well-known spatially averaged thermal and viscous functions. Instead, for a materials having tortuous pore, there is a lack in theoretical description of the thermoacoustic phenomenon.

This paper deals with the performance of a thermoacoustic engine in which a tortuous porous material is used as stack. The spatially averaged thermal and viscous functions are obtained by classical models used to describe the sound propagation inside a porous material. In particular the Johnson–Champoux–Allard model is considered. It requires the knowledge of five parameters instead of the only hydraulic radius used to describe the standard stack having straight pores (e.g. circular, slit or square pores). The physical meaning of these parameters is explained starting from a straight circular pore and modifying, step by step, the shape of the pore until it becomes tortuous.

The proposed functions have been included in the Rott theory and implemented in a numerical procedure. The achieved results are useful to analyse the thermoacoustic performance of a standing wave engine and to understand how the gain factor as well as the viscous and thermal losses inside the stack are affected by the tortuosity. A validation of this procedure is given by comparing the obtained results with ones given by DeltaEC software.

This work can be useful to understand the applicability of tortuous porous materials, such as fibrous material as well as open-cell material, for standing wave thermoacoustic engines.

1. Introduction

Thermoacoustic systems represent new types of devices which are able to convert heat into acoustic energy and viceversa [1]. They can convert low grade energy to acoustic energy in the case of thermoacoustic engines [2] and they use acoustic energy to subtract heat energy in the case of thermoacoustic refrigerators [3]. In this study a thermoacoustic standing wave engine is analyzed. It is a specific kind of heat engine, which basically consists of a porous medium called stack and two heat exchangers inserted into an acoustic resonator. The stack is the key component of these devices influencing their thermodynamic performance. In the stack, the interaction between work fluid and solid occurs so as to generate the gas oscillation with the resulting conversion of heat power into acoustic power. Stacks are usually made by large number of thin plates arranging at equal spacing [1]. Other simple geometries such as circular, square or hexagonal pores (honeycombs) or pin-arrays have also been considered [4]. Afterwards, more complex materials and structures have been studied as Reticulated Vitreous Carbon (RVC) [5,6], stainless steel wire meshes [7] and open-cell porous materials [8]. Other steel porous media have been taken into account to develop low-cost thermoacoustic devices [9]. In the thermoacoustic theory, the stack is characterized by thermo-viscous functions [10]. These functions characterize both heat transfer and dynamics of oscillating flow in porous media. A closed-form solution is possible only for stacks having uniform cross section and simple geometries [4,11,12]. The aim of this work is to analyse the thermoacoustic behavior of stacks having complex geometry by an analytical model used to study the sound propagation inside porous materials. This model assumes that the porous material has a rigid frame and it describes the material as an "equivalent fluid" characterized by a complex density $\check{\rho}$ and a complex bulk modulus $\check{K}$ [13,14]. The complex density $\check{\rho}$ depends mainly on viscous interactions between air inside the pores and the solid skeleton of the porous material and it can be described

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by the Johnson et al. model [15]. The complex bulk modulus $\breve{K}$ depends mainly on thermal interactions and it can be obtained by the Champoux et al. model [16]. The model that makes use of these acoustics parameters, defined in the following as Johnson–Champoux–Allard (JCA) model, depends on five non-acoustic parameters: porosity ϕ, air flow resistivity σ, tortuosity α_∞, thermal characteristic length Λ_t and viscous characteristic length Λ_v. Further improvements of this model due to Pride [17] and Lafarge [18] are not taken into account in this work.

The effects of tortuosity have been taken into account in a previous study by Roh et al. [19]. It extends the thermoacoustic theory valid for parallel capillary-tube to random porous media. In particular a dynamic shape factor, introduced by Stinson, [20] is used. This paper can be seen as a generalization of the above mentioned study. In particular it has the advantage that all the acoustic parameters can be measured or estimated directly without using adaptive parameters.

The influence of the air flow resistivity has been also studied in a previous work for a standing wave device [21].

The physical meaning of the above mentioned parameters and their influence on the thermoacoustic performance are carried out analysing a stack composed by a straight cylindrical pore. The shape of the latter is then modified to obtain a tortuous pore. A Matlab code, based on the Rott's theory [10], that takes into account the new thermal and viscous averaged functions, is built and validated with the available software DeltaEC [22] for stacks having a simple pore shape.

One of the advantages of this analysis is to separate the thermal and viscous losses considering different parameters while in the classical analysis only one parameter can be considered: the hydraulic radius. Besides it is also important to understand the operation frequency at which this kind of stack effectively works. Even if this study is focused on standing wave devices, the same analytical approach can be also used for traveling wave devices to study the thermoacoustic conversion characteristic of regenerator [23].

2. Theoretical basis

2.1. Review of the basic formula

Thermoacoustic behavior of a stack can be studied starting from the momentum and continuity equations, taking into account both viscous and thermal exchanges occurring inside it [1,4]. Using the Swift notation, they can be given by:

$$\frac{dp_1}{dx} = -\frac{j\omega\rho_m}{1-f_v}\langle u_1\rangle \tag{1}$$

$$\frac{d\langle u_1\rangle}{dx} = -\frac{j\omega}{\gamma p_m}[1+(\gamma-1)f_k]p_1 + \frac{(f_k-f_v)}{(1-f_v)(1-P_r)}\frac{dT_m}{dx}\frac{\langle u_1\rangle}{T_m} \tag{2}$$

where p_1 is the acoustic pressure and $\langle u_1\rangle$ is the mean value of the particle velocity evaluated on the cross section of the pore. $j=\sqrt{-1}$, $\omega = 2\pi f$ is the angular frequency and f the frequency, P_r is the Prandtl number and γ is the isobaric and isochoric specific heats ratio. ρ_m, p_m and T_m are respectively the static density, pressure and temperature around which their instantaneous values ρ_1, p_1 and T_1 vary. f_k and f_v are the spatially averaged thermal and viscous functions. In the classical thermoacoustic theory Eqs. (1) and (2) are written in terms of volume flow rate while in this work they are expressed in terms of particle acoustic velocity. These terms differ for the area of the fluid A_{fluid} which is related to the cross section area of the tube A by the porosity ϕ of the stack: $A_{fluid} = A\phi$. In turn A affects the maximum frequency at which the thermoacoustic engine can work because, above this frequency, a plane wave does not propagate in the tube anymore. The maximum frequency f_{max} is related to A by $f_{max} = 0.519c_m/A^{0.5}$ where c_m is the sound velocity in the fluid at the minimum working temperature in order to satisfy this condition also for higher temperatures [24].

The area of the tube is not taken into account because it does not influence temperature, pressure and particle velocity trends inside the stack, therefore it can be seen as a scale parameter that gives the dimension of thermoacoustic engine.

Eqs. (1) and (2) can be written in a more compact form, highlighting thermal and viscous losses:

$$\frac{dp_1}{dx} = -(j\omega l + r_v)\langle u_1\rangle \tag{3}$$

where l and r_v are the inertance and the viscous resistance per unit length of pore and they can be written as:

$$\begin{aligned} l &= \rho_m\frac{1-Re(f_v)}{|1-f_v|^2} \\ r_v &= \omega\rho_0\frac{-Im(f_v)}{|1-f_v|^2} \end{aligned} \tag{4}$$

In the same way, Eq. (2) can be written as:

$$\frac{d\langle u_1\rangle}{dx} = -\left(j\omega c + \frac{1}{r_k}\right)p_1 + g\langle u_1\rangle \tag{5}$$

where c and r_k are the compliance and thermal–relaxation resistance per unit length of pore given by:

$$\begin{aligned} c &= \frac{1}{\gamma p_m}[1+(\gamma-1)Re(f_k)] \\ \frac{1}{r_k} &= -Im(f_k)\frac{\omega(\gamma-1)}{\gamma p_m} \end{aligned} \tag{6}$$

In Eq. (5) compares also the most important term g that represents the gain/attenuation factor whose expression is:

$$g = \frac{(f_k-f_v)}{(1-f_v)(1-P_r)}\frac{dT_m}{dx}\frac{1}{T_m} \tag{7}$$

Solving the system of Eqs. (1) and (2), or what is the same as Eqs. (3) and (5), it is possible to determine the value of p_1 and $\langle u_1\rangle$ inside the stack if the temperature changes of fluid dT_m/dx, in each section of the stack, is known. All terms that refer to physical properties of the fluid depend also on its temperature T_m. Therefore also the temperature variation inside the stack must be taken into account. Its variation can be obtained starting from the equation of total power $\dot{H}$ as follows:

$$\frac{dT_m}{dx} = \frac{\frac{\dot{H}}{A_{fluid}} - \frac{1}{2}Re\left[p\langle\tilde{u}_1\rangle\left(1-\frac{T_m\beta(f_k-\tilde{f}_v)}{(1+\varepsilon_s)(1+P_r)(1-\tilde{f}_v)}\right)\right]}{\frac{\rho_m c_p|\langle u_1\rangle|^2}{2\omega(1+P_r)|1-f_v|^2}Im\left[\tilde{f}_v + \frac{(f_k-\tilde{f}_v)(1+\varepsilon_s f_v/f_k)}{(1+\varepsilon_s)(1+P_r)}\right] - k - \frac{1-\phi}{\phi}k_s} \tag{8}$$

In Eq. (8), the symbol $\sim$ denotes the complex conjugate, β is the thermal expansion coefficient, ε_s is a correction factor for finite solid heat capacity, k and k_s are the thermal conductivity of the fluid and solid phase respectively.

The acoustic power per unit length of the porous medium is also useful for further discussion [1]:

$$\frac{d\dot{E}}{dx} = \frac{1}{2}A_{fluid}Re\left[\langle\tilde{u}_1\rangle\frac{dp}{dx} + \tilde{p}_1\frac{d\langle u_1\rangle}{dx}\right] \tag{9}$$

Replacing the Eqs. (3) and (5) in Eq. (9), it is possible to write this equation in a more convenient form:

$$\frac{d\dot{E}}{A_{fluid}dx} = -\frac{r_v}{2}|\langle u_1\rangle|^2 - \frac{1}{2r_k}|p_1|^2 + \frac{1}{2}Re[g\tilde{p}_1\langle u_1\rangle] \tag{10}$$

The term on the left represents the available acoustic power per unit of area (i.e. acoustic intensity) obtained in an infinitesimal length dx of the stack. On the right of Eq. (10), the first term