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Construction and analysis of an automatic multirate time domain simulation method for large power systems

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ABSTRACT

A multirate method for time domain simulation of large power systems is presented in this paper. Analysis of the proposed multirate method shows how the coupling between fast and slow components affects the numerical stability of the method. Numerical results for two power system examples are presented. Comparison with the corresponding standard integration method shows that substantial gain in computational work and CPU time can be obtained.

1. Introduction

Time domain simulation is one of the key components in the design and operation of power systems. Transient stability simulation tools are frequently used for offline studies and planning, whereas real time simulations are part of dispatcher training simulator engines used for everyday operating tasks and for the emergency situations. Mathematical modeling of power system's time evolution leads to a set of differential and algebraic equations (DAEs) which describe the network, the generators, the voltage regulators, the speed governors and the dynamic loads. Traditional numerical integration methods for DAEs use variable time stepping that starts the solution with a small time step and is gradually increasing it while still satisfying numerical stability and accuracy criteria [3]. In such methods, the size of time step used for the integration of the whole system is chosen to match the dynamics of the most active component.

Large interconnected power systems are modeled by very big DAEs of which some components may exhibit a significantly more active behavior than others, distinguishing slow and rapid temporal variations. Some transients linked to machine fluxes can last a few milliseconds but a secondary frequency control may have a time duration of several minutes. An example of a particular situation is to check the consequences of an outage. In a large power system most of the time the consequences of an outage are very well localized and only a few variables are impacted. In such cases, it is not efficient to simulate the entire system with a small integration time step, since most of the system's variables react slowly and their accuracy constraints can be easily satisfied with a larger step size. Such systems can be efficiently solved using multirate methods. Multirate methods attempt to take large time steps for slowly varying components and small steps for components with a significantly more rapid variation, so as to speed up the numerical computation.

The concept of multirate methods was first introduced by Gear [14]. Later, a study of multirate linear multistep methods was presented in [15]. Multirate methods for explicit methods and non-stiff problems have been examined by Engstler and Lubich [10]. In the proposed method extrapolation was used, and in their strategy the partitioning into different levels of slow to fast components was obtained automatically during the extrapolation process. Multirate methods were successfully applied to the simulation of electric circuits [16,22,32], modular dynamic systems [29], multibody systems [2,23]. In most of these simulations the partitioning into slow and fast components was done in advance before solving the problem, based on knowledge of the DAE system to be solved. First application of multirate methods for power systems was proposed in [8], where a multirate scheme based on trapezoidal rule was introduced. Consistency and stability of this scheme applied to ODEs was analyzed in [9]. Further, a variable partitioning strategy based on the estimation of the local truncation errors and Newton method's residuals was developed in [7]. An alternative multirate strategy for power-electronic-based systems was proposed in [26]. In this strategy partitioning of the components in fast and slow is known in

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advance and does not change in time. A multirate approach which combines Electro-Magnetic Transient (EMT) and Fundamental-Frequency (FF) techniques was proposed in [27]. In this approach, the system is divided in two sub-domains: FF sub-domain and EMT sub-domain. Since EMT simulations focus on small time constant phenomena and FF approximations cover slower phenomena, large time steps are used for the integration of the FF sub-domain and small time steps are used for the integration of EMT sub-domain.

Besides multirate methods, multiple approaches were considered for the time domain simulation of large power systems in the recent years. Localization techniques for dynamic simulation of power systems were presented in [1,13]. During the simulation, the components of the power system are divided into latent and active. Dynamic models of the latent components are replaced by much simpler equivalents, whereas original dynamic models are used for the active components. A variant of the waveform relaxation method for the transient stability analysis of large power systems was presented in [28]. The authors improve the classical waveform relaxation approach by using the initialization of nonlinear integrations with linear solutions and a preconditioning technique to deal with a large number of subsystems. A fully parallel instantaneous relaxation method for real-time transient stability simulation was proposed in [20]. The system is partitioned in a number of subsystems using a coherency-based partitioning. A window length of one time step is used. The proposed instantaneous relaxation method can also be successfully used on multiple graphics processing units (GPUs) [21]. A time domain transformation method for accelerating simulation of power systems is presented in [12]. The authors propose a time domain transformation which allows for use of very large time steps without sacrificing accuracy. The properties of the transformation and the numerical solutions based on the transformation are discussed. The slow coherency decomposition method was proposed in [33]. The authors apply this method for partitioning of all the network buses into coherent areas based on the slow-coherency technique. The primary objective of the proposed slow coherency based partitioning is for use in network reduction. A heuristic partitioning algorithm for parallel processing of large power systems network equations was developed in [5]. The authors present a bordered block diagonal form (BBDF) partitioning algorithm based on matrix orderings and factorization path tree partitioning. A tabu search type of algorithm for partitioning of power system networks was proposed in [6]. The tabu search nature of the algorithm allows for a quick partitioning of the system network into several subsystems which are later solved using parallel computing techniques. Combination of the genetic algorithm and Lagrangian relaxation decomposition techniques were used for generation unit commitment problems in [24]. A particular strength of this combination is the ability to handle both discrete and continuous parameters in the problem formulation. Simulated annealing was used for partitioning of power system networks in [17]. The algorithm the authors considered has a theoretical basis in combinatorial optimization, rather than a heuristic derivation.

In this paper we propose a multirate method based on mixed Adams-BDF scheme. We analyze the numerical stability of the proposed method for both ODE and DAE cases. We also present a construction algorithm for an automatic multirate strategy which includes dynamic partitioning of the differential and algebraic variables based on the topology of the power system and efficient step size management. The main contribution of our paper is that, compared to [9], we analyze the numerical stability of multirate trapezoidal rule scheme for both ODE and DAE test cases and provide the asymptotical stability plots. Moreover, the proposed multirate strategy is based on our numerical stability analysis findings and uses the information about the structure of the power system. Thus, the resulting strategy allows for large speed up in the computational time, maintains the numerical accuracy and avoids the occurrence of instabilities. In order to illustrate the proposed strategy, results for two test problems are presented.

2. Multirate mixed Adams-BDF method

Our multirate method is similar to [18,30] and can be described as follows. For a given global time step, we first compute a tentative approximation for all variables. After this first step, we identify the fast components which need to be computed with smaller time steps in order to ensure the required accuracy. We recompute the values of the fast variables in the refinement stage where we perform a local variable time stepping.

For the formal description of our multirate method, let us assume that the time evolution of a power system is modeled by a non-linear system in semi-explicit form

$$\begin{aligned} y' &= f(t, y, z), \\ 0 &= g(t, y, z), \end{aligned} \tag{1}$$

with initial values $y(0) = y_0$, $z(0) = z_0$, such that $g(0, y_0, z_0) = 0$. Here, t stands for time, y is the vector of differential variables, z is the vector of algebraic variables, f represents the right-hand side of the differential part of the system and g represents the right-hand side of the algebraic part of the system. We also assume that the matrix $\partial g/\partial z$ is non singular and, therefore, system (1) has index one. The joint vector of differential and the algebraic variables is denoted by $u = [y, z]^T$.

System (1) can be rewritten as

$$Ju' = h(t, u), \tag{2}$$

where J is a diagonal projection matrix, with diagonal entries zero or one, where an entry one indicates that the variable is differential and an entry zero that the corresponding variable is algebraic; and $h(t, u) = [f(t, y, z), g(t, y, z)]^T$.

In this paper, for solution of (2) we use the second-order mixed Adams-BDF method presented in [4]. In this method second-order Adams method is applied to the differential state variables, whereas algebraic state variables are integrated using second-order BDF method. The Adams method we use, is symmetrically A-stable (the domain of stability coincides with the left complex half-plane) and thus does not suffer from the hyper stability in contrast to the BDF method. Therefore, if the DAE system is itself unstable, the Adams method will lead to an unstable solution and will allow for detection of instabilities. We use the BDF method for the algebraic state variables, since it is less sensitive to the variations in the algebraic equations than the Adams method.

Let us assume that we know, at time t_n, the numerical approximation of the solution u_n together with its first two derivatives u_n' and u_n'', and we want to compute the numerical solution at time $t_{n+1} = t_n + \tau_n$. We store the vector of the solution and its derivatives in Nordsieck form

$$\mathbf{u}_n = [u_n, \tau_n u_n', 1/2\tau_n^2 u_n'']. \tag{3}$$

The formula for the tentative global step is given by

$$\mathbf{u}_{n+1} = (P \otimes I)\mathbf{u}_n + (l^A \otimes J + l^B \otimes (I - J))((e \otimes I)\mathbf{u}_{n+1} + (eP \otimes I)\mathbf{u}_n)), \tag{4}$$

where $P \in \mathbf{R}^{3\times 3}$ is the Pascal matrix, I is the identity matrix, $e = (1, 0, 0)$, l^A and l^B are the coefficients of the Adams and BDF methods respectively, and $\otimes$ denotes the Kronecker product.

Based on their activity we partition the variables in several disjoint sets (see Section 4). Without loss of generality we assume that the variables are partitioned into two sets of variables: fast and slow variables

$$y = [y_{\text{fast}}, y_{\text{slow}}]^T \quad \text{and} \quad z = [z_{\text{fast}}, z_{\text{slow}}]^T. \tag{5}$$