Applied Thermal Engineering 102 (2016) 227–233

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Research Paper

Natural convection effects in the heat transfer from a buried pipeline

E. Rossi di Schio [a,*], S. Lazzari [b], A. Abbati [a]

[a] Alma Mater Sudiorum – Università di Bologna, Department of Industrial Engineering, Viale Risorgimento 2, I-40136 Bologna, Italy
[b] Università di Genova, Department of Sciences for Architecture, Stradone S. Agostino 37, I-16123 Genova, Italy

HIGHLIGHTS

- Natural convection around a pipe buried in a Darcy porous medium is studied.
- The case of steady state regime is investigated as a particular case.
- The pure conduction case is investigated and compared with the literature.
- The effect of buoyancy is non trivial, for all the considered values of the investigated parameters.

ARTICLE INFO

Article history:
Received 14 October 2015
Accepted 25 March 2016
Available online 1 April 2016

Keywords:
Buried pipelines
Natural convection
Darcy law
Porous medium
Finite element method

ABSTRACT

In the present paper, the effect of the buoyancy on the heat transfer from a buried pipeline is investigated. The soil surrounding the pipe is modelled as a porous medium saturated by water, and the Darcy's law is assumed. Reference is made to a time-varying temperature distribution on the soil surface, and uniform and constant temperature on the pipe wall. The steady state regime is investigated as particular case. The heat power per unit length is evaluated for different values assumed by the Darcy–Rayleigh number, thus revealing that, for high values of *Ra*, the natural convection effects are non trivial. A comparison with the case of pure conduction is made for the limiting case of a vanishing value of *Ra*, revealing an excellent agreement.

1. Introduction

Heat transfer from buried pipes and cables has been widely investigated in the literature because it is crucial in many engineering applications [1–9]. In the above mentioned papers, the typical approach to the problem of determining heat transfer is to model the soil as a purely conducting solid. Indeed, the problem investigated is that of heat conduction in a semi-infinite conducting medium around a cylinder. Interesting results are presented in [7], where an analytical expression of the steady-state heat transfer coefficient from an offshore buried pipeline to its environment can be found. The analysis refers to the boundary condition of a uniform temperature of the seabed, i.e. of the separation surface between soil and sea water. More recently, the analytical expression of the heat power exchanged between soil and pipe has been evaluated by Barletta et al. [8] in order to consider a boundary condition on the soil given by a sinusoidal function of time.

In the literature, some papers investigate the natural convection in a porous soil surrounding a cylinder. The point of view is not necessarily strictly concerning the applications of buried pipes and cables, and in some cases the analyses refer to an infinite medium surrounding the cylinder. Farouk and Shayer [10] investigate numerically the natural convection heat transfer from a heated cylinder buried in a semi-infinite porous medium saturated by water. Reference is made to the steady state regime and the upper boundary is considered as a permeable surface. Chang et al. [11] investigate the natural convection in a tube partially filled with a porous medium. Himasekhar and Bau [12] present an analytical and numerical study on the natural convection from horizontal hot/cold pipes buried in a semi-infinite saturated porous medium. Fand et al. [13] investigate the natural convection heat transfer from a horizontal cylinder embedded in a fluid saturated porous medium, modelled by considering both Darcy's law and Darcy's–Forchheimer law. In [14] an experimental and theoretical analysis of the steady heat convection around a heat source embedded in a box containing a saturated porous medium is presented. Bau and Sadhal [15] analyses the convective heat losses from a pipe buried in a semi-infinite porous medium and Facas [16] extends the work presented in [15] for a case with baffles attached to the cylinder. In

* Corresponding author.
E-mail addresses: eugenia.rossidischio@unibo.it (E. Rossi di Schio), stefano.lazzari@unige.it (S. Lazzari), alessia.abbati@studio.unibo.it (A. Abbati).

http://dx.doi.org/10.1016/j.applthermaleng.2016.03.140
1359-4311/

Nomenclature

g	modulus of $\mathbf{g}$ [m s^{-2}]
$\mathbf{g}$	gravitational acceleration [m s^{-2}]
k	effective thermal conductivity [W/(m K)]
K	permeability [m^2]
l	dimensionless length of the pipeline circumference
$\vec{n}$	unit outward-pointing vector normal to the boundary surface
$\dot{Q}$	dimensionless power per unit length, Eq. (10)
Ra	Darcy–Rayleigh number, Eq. (3)
R_0	radius of the pipe [m]
t	dimensionless time, Eq. (3)
T	dimensionless temperature, Eq. (3)
T_w	pipe boundary temperature [K]
T_0	mean value of the boundary temperature [K]
u, v	Cartesian components of the dimensionless seepage velocity, Eq. (3)
x, y	dimensionless Cartesian coordinates, Eq. (3)

Greek symbols

α	effective thermal diffusivity [m^2 s^{-1}]
β	volumetric coefficient of thermal expansion [K^{-1}]
γ	dimensionless burying depth, Eq. (3)
ΔT	amplitude of the boundary temperature oscillation [K]
Λ	dimensionless parameter, Eq. (8)
σ	heat capacity ratio
ω	pulsation [s^{-1}]
Ω	dimensionless parameter, Eq. (8)

Mathematical symbols

∇	dimensionless del operator, Eq. (3)

Superscript, subscripts

$\bar{\ }$	dimensional quantity

[17] the steady, two-dimensional, free convection around line heat sources and heated cylinders in unbounded saturated porous media is presented.

An analysis of the above mentioned papers shows that most of them refer to steady regime. In the present paper we aim to investigate the unsteady two-dimensional convection heat transfer from an isothermal cylinder buried in a semi-infinite porous medium saturated by water. The thermal boundary condition prescribed on the soil surface is a time-varying temperature distribution expressed by a sinusoidal function, and particular attention is paid to the role of the buoyancy effects.

2. Mathematical model

Let us describe the soil surrounding a buried pipeline as a semi-infinite porous medium saturated by water, bounded at $\bar{y} = H$ by a surface kept at the following time-dependent temperature distribution:

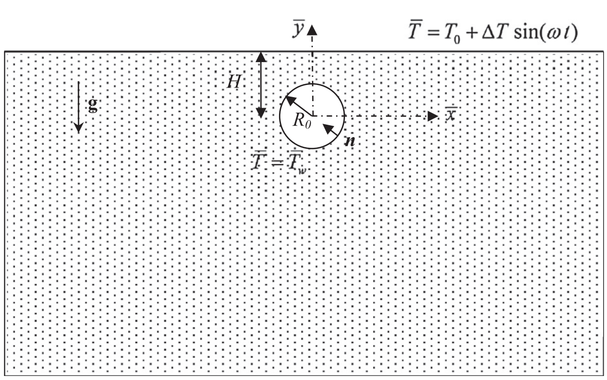

Fig. 1. Drawing of the system.

$$\bar{T} = T_0 + \Delta T \sin(\omega t). \tag{1}$$

H is the burying depth of the pipe, defined as the distance between the soil surface and the axis of the pipe. Let us assume that the vertical axis $\bar{y}$ is parallel to the gravitational vector $\mathbf{g}$, but with opposite direction. Since pipes are usually much longer than wide, the problem under consideration can be approximated as two-dimensional, as sketched in Fig. 1. The pipeline section is circular, with radius R_0, and its wall is kept at the uniform and constant temperature T_w. Indeed, we are interested in studying heat transfer from the external surface of the pipe to the soil, i.e. we are not solving for the fluid flow inside the pipe. Let us assume that the Darcy's law holds, the thermophysical properties are constant and the Boussinesq's approximation can be applied.

On account of the above assumptions, the local mass, momentum and energy balance equations in the soil domain are, respectively,

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0,$$

$$\frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{g\beta K}{\nu}\frac{\partial \bar{T}}{\partial \bar{x}},$$

$$\sigma\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\bar{\nabla}^2\bar{T}, \tag{2}$$

where the rotational has been applied in the momentum equation.

In order to generalize the study, let us introduce the following dimensionless quantities:

$$x = \frac{\bar{x}}{R_0}, \quad y = \frac{\bar{y}}{R_0}, \quad \nabla = R_0\bar{\nabla}, \quad u = \frac{\bar{u}R_0}{\alpha}, \quad v = \frac{\bar{v}R_0}{\alpha},$$

Table 1
Dimensionless heat power per unit length for $\gamma = 2, \Gamma = 0, Ra = 0$: analysis of the different computational domains.

Domain size	$\dot{Q}$
$100 \times 100 + \gamma$	4.7637
$200 \times 100 + \gamma$	4.7690
$200 \times 200 + \gamma$	4.7690
$400 \times 200 + \gamma$	4.7702

Table 2
Dimensionless heat power per unit length for $\gamma = 2, \Gamma = 0, Ra = 0$: analysis of the different grids.

Grid elements	$\dot{Q}$
$6968 + \gamma$	4.7661
$27{,}872 + \gamma$	4.7694
$111{,}488 + \gamma$	4.7702