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Improvement of vector fitting by using a new method for selection of starting poles

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ABSTRACT

Vector fitting (VF) is known as the most popular method for frequency-dependent modelling. Using VF, frequency-domain data of a network can be converted into an equivalent circuit model. Some initial poles need to be selected before VF can be started. The starting poles affect the accuracy and convergence speed of the vector fitting method. In this paper, a new procedure is proposed to select starting poles. The proposed procedure selects starting poles by partitioning the frequency response, and then ranks the starting poles so that the most dominant poles can be used when low order approximation is desired. Case study results show that the proposed procedure improves the accuracy and the speed of VF significantly.

1. Introduction

The study of power quality often requires the detailed modelling of complex networks. However, detailed representations may lead to an excessive computation burden. A common practice to reduce the computational burden is to divide a system into a study zone and an external system encompassing the rest of the system [1]. The external system is commonly represented by a Frequency-Dependent Network Equivalent (FDNE) circuit [2,3]. Using equivalent circuit models in a general simulation environment is straightforward, enabling fast simulation in both time and frequency domain [2,4]. The equivalent network should be developed based on the frequency domain data of the system. The frequency domain data is the tabulated data including impedances and voltage harmonics for different frequencies. This data can be generated by using measurements or power quality simulation tools.

Vector fitting (VF) with its several formulations [3–10] is the most commonly used method for frequency-dependent modelling. Although new methods based on evolutionary computation such as Refs. [11–13] have been recently proposed, VF is still the most popular method due to its simplicity and robustness. VF gets the given frequency-response data and finds a rational function approximation with guaranteed stability. The identified rational function can then be converted to an equivalent electric circuit model using the algorithm proposed in Ref. [4].

To start the vector fitting method, some initial poles should be selected. The initial poles strongly affect the accuracy and convergence speed of the vector fitting method. The most commonly used procedure for the selection of starting poles in the one proposed in Ref. [3]. This procedure suggests that the starting poles should be complex conjugate with the imaginary parts linearly or logarithmically distributed over the frequency range of interest. By using this method, however, the starting poles might be very far from the actual poles. This will result in two problems:

- A high order approximation with a large number of iterations may be required to achieve an acceptable accuracy.
- In practical applications, one often wants to use a low order approximation for a high order function. In this case, the method might not be able to find the most dominant poles, resulting in a larger approximation error.

The above problems can be significantly magnified when the frequency response contains noise. Experience with the VF algorithm has shown that the existence of noise can significantly impair convergence, leading to possibly inaccurate models due to the presence of noise-induced spurious poles [16]. As discussed in Ref. [17], this problem is also related to the pole relocation of VF which may fail to reach the optimum poles. During the pole relocation, the poles may be relocated in small steps and the convergence may even stall. Noise-induced spurious poles may be identified by VF instead of actual dominant poles, resulting in inaccurate models.

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This paper aims to alleviate the above problems by proposing a new procedure for starting pole selection. The proposed procedure selects starting poles by partitioning the frequency response, and then ranks the starting poles so that the most dominant poles can be used when low order approximation is desired. The detailed concept and algorithms of the proposed procedure are presented in this paper. The effectiveness of the procedure is also confirmed using different case studies.

2. Vector fitting method

The aim of vector fitting is to find a rational function that approximates the frequency-domain data of the system. VF is an iterative technique based on pole-zero relocation technique. This method is reviewed in this section.

The frequency response $G(s)$ can be represented using rational functions. For an Nth order system, the rational function can be written as:

$$G(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - p_n} + d + se \tag{1}$$

where the residues cn and poles pn are either real quantities or come in complex conjugate pairs, while e and d are real, and the problem is to estimate all coefficients in Eq. (1). Clearly, the optimization problem is non-linear in the poles p_i. To solve this problem, Gustavsen and Semlyen [3] propose to identify the parameters indirectly. For this purpose, two other transfer functions $\sigma(s)$ and $H(s)$ are defined as follows:

$$\sigma(s) = \sum_{n=1}^{N} \frac{\widehat{c}_n}{s - \widehat{p}_n} + 1 \tag{2}$$

$$H(s) = \sigma(s)G(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - \widehat{p}_n} + d + se \tag{3}$$

$H(s)$ has the same structure as $G(s)$ and $\sigma(s)$ has a unit gain in high frequencies. As seen in Eqs. (2) and (3), $G(s)$ and $\sigma(s)$ have identical poles. These poles are supposed to be known at the beginning of each iteration.

2.1. Identification of parameters in $\sigma(s)$ and $H(s)$

By substituting Eq. (2) in Eq. (3), we will have:

$$\left(\sum_{n=1}^{N} \frac{\widehat{c}_n}{s - \widehat{p}_n} + 1 \right) G(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - \widehat{p}_n} + d + se \tag{4}$$

Eq. (4) can be re-written as follows:

$$\left(\sum_{n=1}^{N} \frac{c_n}{s - \widehat{p}_n} + d + se \right) - \left(\sum_{n=1}^{N} \frac{\widehat{c}_n}{s - \widehat{p}_n} \right) G(s) \approx G(s) \tag{5}$$

In matrix forms, it can be rewritten as:

$$\begin{bmatrix} \frac{1}{s_1 - \widehat{p}_1} & \cdots & \frac{1}{s_1 - \widehat{p}_N} & \frac{-G(s_1)}{s_1 - \widehat{p}_1} & \cdots & \frac{-G(s_1)}{s_1 - \widehat{p}_N} & 1 & s_1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{1}{s_k - \widehat{p}_1} & \cdots & \frac{1}{s_k - \widehat{p}_N} & \frac{-G(s_k)}{s_1 - \widehat{p}_1} & \cdots & \frac{-G(s_k)}{s_1 - \widehat{p}_N} & 1 & s_k \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_N \\ \widehat{c}_1 \\ \vdots \\ \widehat{c}_N \\ d \\ e \end{bmatrix} = \begin{bmatrix} G(s_1) \\ \vdots \\ \vdots \\ G(s_k) \end{bmatrix} \tag{6}$$

where k is the number of data points in the frequency-domain data.

As explained before, $\widehat{p}_i$, the starting poles, are known. Therefore, Eq. (6) is a linear equation with respect to $\widehat{c}_n$, c_n, d and e. Thus, the least square method can be employed to obtain these parameters. By identification of these parameters, $\sigma(s)$ and $H(s)$ are determined.

2.2. Identification of poles in $G(s)$

Transfer functions $H(s)$ and $\sigma(s)$ can be written as:

$$H(s) = \sigma(s)G(s) = \frac{\prod_{n=1}^{N+1}(s - z_n)}{\prod_{n=1}^{N}(s - \widehat{p}_n)} \tag{7}$$

$$\sigma(s) = \frac{\prod_{n=1}^{N}(s - \widehat{z}_n)}{\prod_{n=1}^{N}(s - \widehat{p}_n)} \tag{8}$$

Using Eqs. (7) and (8), $G(s)$ can be written as follows:

$$G(s) = \frac{\sigma(s)G(s)}{\sigma(s)} = \frac{\prod_{n=1}^{N+1}(s - z_n)}{\prod_{n=1}^{N}(s - \widehat{z}_n)} \tag{9}$$

Eq. (9) indicates that poles of $G(s)$ are identical with the zeros of the estimated $\sigma(s)$. It should be noted that the starting poles are cancelled out because the same poles are used to estimate $H(s)$ and $\sigma(s)$. Since $\sigma(s)$ has been identified in the previous step, the poles of $G(s)$ which are equal to the poles of $G(s)$ are easily found.

2.3. Stability of the model

If during the calculation of poles of $G(s)$, some unstable poles are found, they should be made stable in order to ensure $G(s)$ stability. At this point, unstable poles, poles with a positive real part, should be modified to be stable poles with negative real parts. This can be easily done by changing the sign of the real part of the unstable poles [14]. This procedure ensures the stability of the approximated function.