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Research Paper

Design of a longitudinal cooling fin with minimum volume by a modified Newton–Raphson method

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ABSTRACT

In this paper, the minimal volume of nonlinear longitudinal cooling fin design problem by using a modified Newton–Raphson method is presented. The profile of the fin is built by B-spline curve in which the control points of the B-spline curve are regarded as optimization variables. Additionally, a mechanism called "volume updating" is added into the modified Newton–Raphson algorithm to obtain the minimum volume of the fin. Four cases with the different boundary conditions and thermal properties of the longitudinal fin are presented to demonstrate the proposed method. The results show that the optimal fin obtained by the proposed method is in good agreement with Schmidt's (1926) result and is better than that of Azarkish, et al. (2010). It is concluded that the B-spline with the second degree and three control points could be used enough to find the minimum volume of the longitudinal fin for the linear and nonlinear fin design problems. From the results obtained in four cases, it appears that the proposed method is an efficient and accurate method in finding the minimum volume of the nonlinear longitudinal cooling fin design problem.

1. Introduction

Fin is one of the most important pieces of equipment to increase the rate of heat transfer in the heat exchangers. Various types of fins such as longitudinal, spines and radial fins are designed and used for different purposes. They are widely applied in many industries for cooling electronic devices, automobile engine, heat exchangers, and so on. However, when fins are added into a system, this will lead to an increase in the volume of that system. To deal with this issue, in the past few decades, many scientists have been attempted to optimize the fin geometry [1]. Originally, there were two approaches for the optimization analysis of fins. In the first approach, the optimal fin with a given profile like rectangular, triangular, or trapezoidal is obtained from the optimal standard. In the second one, the shape of a fin is determined in order to either minimize its volume for a given heat transfer rate or maximize the heat transfer rate for a given volume. For this second approach, Schmidt [2] suggested that the minimum volume of the optimal longitudinal fin is a concave parabolic profile. He showed that this profile has minimum volume with maximum heat loss, and the temperature distribution along this fin is linear. This result was evaluated later by Duffin [3], employing some powerful formulations by using various methods. In this research, the convective heat transfer coefficient was assumed constant.

Indeed, Ghai [4] presented the experimental and theoretical results that the convective heat transfer coefficient varies significantly along with the surface of fins. Razelos and Imre [5] used Pontryagin's maximum principles to obtain the solution of minimum mass of fins with variable convective heat coefficients. The obtained results showed that under the specific cases of a power law variation of the heat transfer coefficient, mass of fins was two to three times larger than that of constant heat transfer coefficient. Afterward, Natarajan and Shenoy [6] optimized the shapes of pin fin with variable heat transfer coefficient. By the realistic assumption of the variable heat transfer coefficient of $h = 1/D^n$, an important result in [6] showed that Schmidt's criterion of a linear temperature profile holds for optimal convection pin fin of given weight with the variable heat transfer coefficient. In addition, by using Pontryagin's maximum principles, Natarajan also demonstrated that problems of minimizing weight for a given heat transfer rate and maximizing heat transfer rate for a given fin weight are identified if both of them are governed by the same optimal design equations. For convective and radiative fins, a thorough study of the optimal design of fins have been shown in [1]. Also, Malekzadeh, et al. [7] maximized the heat transfer rate for a given volume of the convective and radiative fins by using a combination of the differential quadrature element method. Arslanturk [8] was present a simple correlation equations for optimal design of annular fins with uniform cross section. These optimal results were obtained by using the one-dimensional model. However, two-dimensional analysis can be applied to a wider range of shapes with high accuracy.

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For two-dimensional analysis, Kang and Look [9] estimated the optimal dimension of annular fins with rectangular profile under thermally asymmetric convective and radiating condition by using the two-dimensional model. A comparison between one-dimensional and two-dimensional analysis also presented in [9]. The result showed that the difference between 1-D and 2-D results rises as the fin height increases or as the fin top convection characteristic number increases. Recently, Kang and Look [10] also analyzed and optimized an annular trapezoidal fin by using a new approach for a two-dimensional analytical method. Nonetheless, these optimal results are only obtained by the analytical analysis with the regular shape of fins under the assumption of constant thermal parameters and the variable convective heat transfer coefficient with the special case of the power law.

The drawback of the above analytical methods is that the optimization of the nonlinear fin design problem is difficult when the two-dimensional heat transfer model is investigated. To overcome this limitation, some researchers used the numerical analysis combining with an optimization algorithm to deal with the optimization of the nonlinear fin design problems with the complicated shape [11–14]. Fabbri [11] used the finite element method to estimate the two-dimensional temperature distribution on the longitudinal section of polynomial profile fins. The genetic algorithm (GA) was then applied to optimize the fin effectiveness. Copiello and Fabbri [12] optimized the heat loss from longitudinal wavy fin under forced convection condition. By using multi-objective genetic algorithm, the heat transfer rate and fin efficiency were optimized simultaneously. Also, Hajabdollahi, et al. [13] optimized the one dimensional heat transfer in a pin fin. In the work of Hajabdollahi, et al. [13], Bezier curves is used to construct the shape of pin fin and GA was applied to determine the optimal geometry of the fin. Recently, Azarkish, et al. [14] maximized heat loss for a given volume of longitudinal fin with one-dimensional model. B-spline was used to represent the fin shape, and a modified genetic algorithm was employed to optimize the fin, under the natural convection and radiation condition with volumetric heat generation.

From these papers, it can be seen that GA could be used for the complicated fin design problem with many real conditions. However, GA known as non-gradient based has some weaknesses, one of its drawbacks is the high computational cost. The method is acceptable for the single objective function and one-dimensional model, however, the computational cost significantly rises for complex problems. In contrast, gradient based methods such as Levenberg–Marquardt Method (LMM), Conjugate gradient method (CGM), and Sequential quadratic programming method (SQP) have advantages. These have been used to tackle the optimization of complicated geometry problems with the fast convergence. For instance, Huang and Chen [15] applied LMM to determine non-uniform fin heights and widths for an impingement heat sink module. Bobaru and Rachakonda [16] used SQP for optimization of fin profile with the two-dimensional analysis under the natural convection, and Huang and Hsiao [17] optimized the shape of spine and longitudinal fins based on desired fin efficiency and fin volume by utilizing CGM.

In all the citations above, none of previous published papers proposes the effective and accurate method to minimize the fin volume for nonlinear fin design problems. In this paper, the minimal volume of nonlinear longitudinal fin design problem by using a modified Newton–Raphson method is presented. The finite element method is applied to solve the two-dimensional nonlinear governed heat transfer equation. On the other hand, the fin profile is built by the B-spline curve which is controlled by the control points. The modified Newton–Raphson method is used to find the optimal position of these control points by minimizing an appropriate function representation.

The modified Newton–Raphson method has been applied successfully to solve the inverse problem by Yang [18–20] and Nguyen and Yang [21], and B-spline curves have been used to optimize geometry design. The goal of the optimization problem in this paper is to minimize the volume of the longitudinal fin for a given heat loss and the specified base temperature. Thus, a mechanism called "volume updating" is added into modified Newton–Raphson algorithm to obtain the minimum volume of the longitudinal fin. This approach is based on "curve fitting" mechanism of the modified Newton–Raphson method. In this mechanism, the obtained solution is the best approximation which is defined as that which minimizes the sum of squared differences between the computed and expected value. The results show that the proposed method is efficient and accurate method to minimize the volume of the longitudinal fin for the nonlinear fin design problems.

2. Problem statement

Consider a longitudinal symmetric fin model that dissipates a given amount of heat q_{flow} from the wall at the given temperature, T_b, to the ambient quiescent fluid, T_a. The fin geometry is determined by the semi-profile B-spline curve as shown in Fig. 1. In the steady state condition, the two dimensional heat transfer equation is expressed as following:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=0 \qquad \text{in domain } \Omega \tag{1a}$$

$$-kA_b\frac{\partial T}{\partial x}=q_{flow} \qquad \text{in boundary } \Gamma_0 \tag{1b}$$

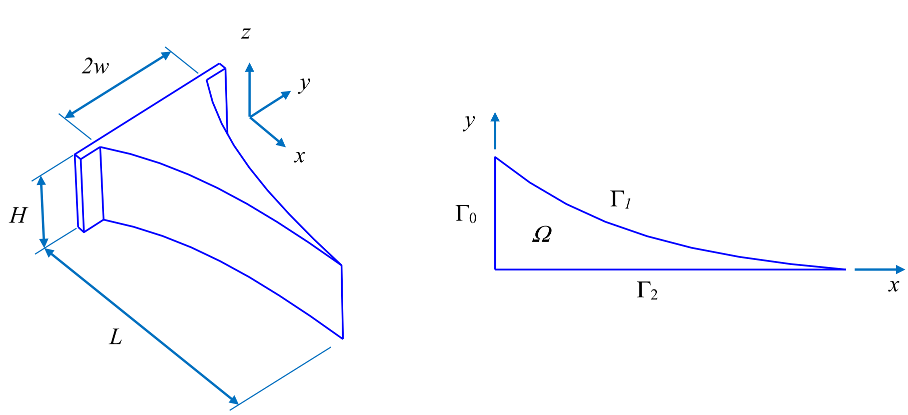

Fig. 1. The 3D model (left) and semi-cross-section (right) of a single longitudinal fin with variable cross section area.